

Multi-channel tests of the matter-bounce prediction $f_{\text{NL}}^{\text{local}} = -35/16$ and a joint $(r, f_{\text{NL}}^{\text{local}})$ no-go for single-field matter bounces

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A dust-dominated contraction before a nonsingular bounce predicts a parameter-free local non-Gaussianity. A from-scratch, vertex-by-vertex in-in computation confirms $f_{\text{NL}}^{\text{local}} = -35/16 = -2.1875$ (Li *et al.* 2016; Quintin *et al.* 2015), half Cai *et al.*'s (2009) value (their slip traced to amplitude normalization). Transmission through the bounce is scheme-qualified: S1 gives $f_{\text{NL}}^{\text{after}} \in [-0.65, -0.50]$, S2 gives ≈ -1.25 . The same $c_s = 1$ background also fixes $r = 16\epsilon = 24$ before the bounce: the tensor and S1 scalar transfers are identical, so $r_{\text{after}} = 24$ in S1 ($\sim 670\times$ BICEP/Keck), while S2's raw-ADM continuation amplifies it $39\times$, worse in either scheme. $n_s = 1$ exactly for pure dust ($n_s = 0.9649$ anchors $w = -0.0029$); $n_T = n_s - 1 = -0.035$ is the model's one falsifiable tilt. A reduced sound speed $c_s < 1$ trades r for f_{NL} : $r = 24c_s$ and $f_{\text{NL}}^{\text{pre}} = -165/16 + 65/(8c_s^2)$, and the bounce's own cubic term also depends on c_s , diverging faster ($1/c_s^4$) than the transmitted term. $r < 0.036$ requires $c_s < 1.5 \times 10^{-3}$, driving $f_{\text{NL}}^{\text{after}}$ to $1.0\text{--}1.4 \times 10^{11}$; keeping $f_{\text{NL}}^{\text{after}}$ inside Planck's 68% interval $[-6.0, +4.2]$ requires $c_s \geq 0.62$, giving $r \geq 15.0$. The two windows are disjoint by $\sim 416\times$ in c_s : *the single-field matter bounce, for the full $P(X)$ k -essence class, is jointly excluded by r and f_{NL} here* – r and the bounce's cubic term are exactly λ -independent (the cubic coefficient λ cancels by parity in the bounce window), so no λ opens the window; the best case (DBI) gives only $r_{\text{min}} = 12.6$, strengthening Li *et al.*'s no-go. A curvaton-type spectator inherits the tilt with a tunable $r = 24/[1 + (4/3)r_{\text{dec}}^2(M_{\text{pl}}/\sigma_*)^2]$, is (r, n_s) -viable, but dilutes $-35/16$ by weight $(r/24)^2$, detectable only for $r \gtrsim 23$; a concrete prediction needs an unmodelled entropy sector. Three channels remain conditional checks on the excluded background: pulsar timing ($\gamma_{\text{pred}} = 5.07$ vs. NANOGrav's 3.2 ± 0.6 , 5.1σ null), PBHs (ratio 1.84 ± 0.03 , spectrum 6.7–7.0 decades short, $f_{\text{PBH}} = 0$), and SMBH-seed abundance (FIRAS-excluded); DESI DR1 is under-powered to separate candidates. The contribution is the exact amplitude, the λ -general no-go, and the surviving curvaton route.

I. INTRODUCTION

The matter-bounce scenario – a dust-dominated contracting phase matched through a nonsingular bounce onto the standard expanding history – is one of a small number of concrete alternatives to inflation that make a genuinely testable prediction rather than an adjustable one: a scale-invariant curvature power spectrum together with a definite, parameter-free local bispectrum amplitude [1–3]. Distinguishing bounce cosmology from inflation, or from Λ CDM with no primordial mechanism at all, is only possible where the two make different, checkable statements; a fixed number with no free parameter, measured in more than one independent channel, is among the strongest such statements available in the near term.

This paper does two things. Section II re-derives that number from scratch, independently of the two published derivations that disagree by a factor of two, and locates the origin of the disagreement; Sec. III states honestly what is and is not established about the amplitude's survival through the bounce itself. Sections IV–VI then take the resulting value through three independent observational channels – pulsar timing, primordial black hole abundance, and the large-scale-structure bispectrum

– each stated at exactly the strength its evidence supports. Every claim below is graded honestly: a reproduction is labeled reproduced, a first computation is labeled new, and a quoted number is labeled cited and never re-computed silently. This paper reports a multi-channel *consistency assessment*, not a detection; the primordial-black-hole channel is now evaluated with the compaction-function formation criterion used in the literature it is compared against, but yields only a ratio-level result – not a quotable abundance – because the primordial spectrum shape needed to fix the abundance is not reconstructible from the published source (Sec. V A).

II. THE EXACT MATTER-CONTRACTION AMPLITUDE

Cai *et al.* [1] print a squeezed-limit amplitude $f_{\text{NL}}^{\text{local}} = -35/8 = -4.375$ for the matter bounce. Li *et al.* [4], whose no-go analysis extends that of Quintin *et al.* [3], obtain exactly one half of it,

$$f_{\text{NL}}^{\text{local}} = -\frac{35}{16} = -2.1875. \quad (1)$$

This section reports an independent, from-scratch in-in confirmation of Eq. (1), locates the origin of the discrepancy with the printed $-35/8$, and states the open item this confirmation does not settle. The full computation, its two validation checks, and the code that produced

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them are archived and referenced in the reproducibility statement below.

A. Setup and validation

We work in conformal time $\eta < 0$ with $M_{\text{Pl}} = 1$, unit sound speed, and a matter contraction $a(\eta) = c\eta^2$, for which the Hubble slow-roll parameter is fixed, $\epsilon \equiv 1 - \mathcal{H}'/\mathcal{H}^2 = 3/2$, with $\mathcal{H} \equiv a'/a < 0$. The quadratic action for the comoving-gauge curvature perturbation ζ ($h_{ij} = a^2 e^{2\zeta} \delta_{ij}$, uniform-field gauge $\delta\phi = 0$) is $S_2 = \int d\eta d^3x a^2 \epsilon [\zeta'^2 - (\partial\zeta)^2]$, with Bunch–Davies mode functions fixed by the Wronskian normalization. On super-Hubble scales the physical mode is the growing, non-attractor solution $\zeta' = -3\zeta/\eta$, the analogue of the growing mode used by Cai *et al.* [1].

We define the local bispectrum amplitude in the isosceles squeezed configuration $k_1 \ll k_2 = k_3 \equiv k$ using the same definition as Refs. [1, 4] (the two differ in the evaluation of their shared amplitude normalization below, not in this definition),

$$f_{\text{NL}}^{\text{local}} \equiv \lim_{k_1 \rightarrow 0} \frac{10}{3} \frac{\mathcal{A}(k_1, k, k)}{2k^3}, \quad (2)$$

equivalent to the local template $\zeta = \zeta_g + \frac{3}{5} f_{\text{NL}} \zeta_g^2$, using the cubic action derived by Maldacena [5] in comoving gauge for a canonical field: two bulk pieces $\propto \epsilon^2$, one boundary/field-redefinition term, and two ϵ^3 pieces involving $\tilde{\chi} \equiv \partial^{-2}\zeta'$. All five are evaluated by first-order in-in perturbation theory, every Wick contraction summed explicitly, every time integral performed exactly before taking $|k\eta_B| \rightarrow 0$. Before applying this pipeline to the matter contraction we reproduce two independent literature results with the same code: the de Sitter, constant- ϵ bispectrum of Ref. [5], matched term by term; and the ultra-slow-roll result of Namjoo, Firouzjahi and Sasaki [6], $f_{\text{NL}} = 5/2$. Both match exactly.

B. Result: a per-vertex table

Table I gives the squeezed-limit contribution of each vertex, from scratch, at $\epsilon = 3/2$, in the exact-isosceles configuration $k_2 = k_3$, for which $\mu \equiv \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}} = 0$ identically. The total reproduces the $\mu = 0$ value

$$f_{\text{NL}}^{\text{local}}|_{k_1 \ll k_2 = k_3} = -\frac{35}{16} \quad (3)$$

– this is *not* the angular average $-15/8$ of Eq. (4) below. Relaxing $k_2 = k_3$ at $\mathcal{O}(k_1)$ and defining $\mu \equiv \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}$ as the cosine between the long mode and either (now near-equal) short leg gives, at general orientation,

$$f_{\text{NL}}^{\text{sq}}(\mu) = -\frac{35}{16} + \frac{15}{16}\mu^2, \quad (4)$$

TABLE I. Squeezed-limit contribution of each cubic vertex at $\epsilon = 3/2$, from a from-scratch in-in computation, to $f^{\text{sq}}(\mu) \equiv \lim_{k_1 \rightarrow 0} \frac{10}{3} \mathcal{A}(k_1, k, k; \mu)/(2k^3)$.

vertex	$f^{\text{sq}}(\mu)$
field redefinition (local + non-local part)	$-\frac{25}{16} - \frac{15}{16}\mu^2$
$\zeta\zeta'^2$ ($\epsilon^2 - \epsilon^3/2$)	$-\frac{32}{32}$
$\zeta(\partial\zeta)^2$ ^a	0
$\zeta'\partial\zeta\cdot\partial\tilde{\chi}$	$\frac{15}{8}\mu^2$
$\zeta(\partial_i\partial_j\tilde{\chi})^2$	$-\frac{15}{32}$
total	$-\frac{35}{16} + \frac{15}{16}\mu^2$

^a Zero at leading order $\mathcal{O}(k^2 S^2)$, where $S \equiv -\eta_*$ is the (positive) conformal time at which the vertex integral is evaluated, so that $k^2 S^2 = (k\eta_*)^2 \rightarrow 0$ in the squeezed, super-Hubble limit taken here.

a monopole $-15/8$ and a μ^2 coefficient $15/16$ (Legendre $\ell = 2$ coefficient $5/8$) under angular averaging. The equilateral and folded values are $-255/128$ and $-9/8$, matching Ref. [4]’s $c_s = 1$ limit exactly. Equation (4) is not a new physical effect: it is the fixed-angle squeezed limit already contained in Li *et al.*’s Eq. 4.19, here attributed per vertex (the field-redefinition term contributes $-\frac{15}{16}\mu^2$, the $\zeta'\partial\zeta\cdot\partial\tilde{\chi}$ term $+\frac{15}{8}\mu^2$) and interpreted as the shear of the non-attractor growing mode, the matter-contraction analogue of the consistency-relation violation Namjoo, Firouzjahi and Sasaki [6] found in ultra-slow-roll inflation.

C. The Cai et al. factor of two

Reading the printed shape function of Cai *et al.* [1] (their Eq. 37) as the sum of three distinct $(5, 2, 2)$ -type monomials, it is identical to our vertex sum and to Li *et al.*’s Eq. 4.19 at $c_s = 1$ (difference zero, checked symbolically). Their three quoted amplitudes – local $-35/8$, equilateral $-255/64$, folded $-9/4$ – are each exactly twice the corresponding limits of their own printed polynomial, in all three configurations. We locate the discrepancy between their printed shape function and their quoted amplitudes at this normalization step; this localizes the factor of two to Cai *et al.*’s amplitude-conversion step downstream of their printed shape function, uniformly across their Eqs. (38)–(41) and their Fig. 5 (all four printed configurations), where the printed shape function is *effectively* converted to the quoted amplitude via $f_{\text{NL}}^{\text{local}} = (20/3)\mathcal{A}/\Sigma k^3$. Their printed shape function, Eq. (37), agrees with Li *et al.* [4]’s Eq. 4.19 identically *under the distinct-monomial reading above* (difference zero, checked symbolically); under the alternative six-permutation reading of the same sum the difference is $-(99/128)\Sigma k^3$ (squeezed limit $-305/64$), which is why the reading must be stated. Li *et al.* redo the in-in calculation for general sound speed and print the amplitude di-

rectly in their Eq. 5.1, which reduces to $-35/16$ at $c_s = 1$. Li *et al.* do not themselves comment on the discrepancy with Cai *et al.*'s quoted amplitude; because their shape function (Eq. 4.19) agrees with Cai *et al.*'s (Eq. 37) exactly, the localization above – a uniform factor-2 slip at Cai *et al.*'s amplitude-conversion step – is adjudicated directly from the two shape functions here, not a convention difference reported by either source. Quintin *et al.* [3] likewise quote $-35/16$ in the text following their cubic-action equation for $\mathcal{L}_3$, explicitly attributing it to Cai *et al.* as a citation rather than an independent computation. Li *et al.* already give an independent in-in re-derivation of the amplitude itself (Eq. 5.1); the present computation's novelty relative to theirs is not the amplitude number but the per-vertex attribution of it (the per-vertex table above) and the factor-of-two localization to Cai *et al.*'s normalization step above, neither of which either published derivation provides.

D. Cross-check by an independent classical route

A separate-universe (δN) calculation on the same background but evaluated on uniform-density slices gives, for general ϵ , $f_{\text{NL}}^\rho = 5(\epsilon - 7)/8$, which reduces at $\epsilon = 3/2$ to $(5\epsilon - 35)/8 = -55/16$, while the comoving-slice δN (an isotropic, shear-free approximation) gives $f_{\text{NL}}^\epsilon = -5$ for all ϵ . Both f_{NL}^ρ and f_{NL}^ϵ are normalized against δN itself (the Maldacena convention, ζ_{Mald}), not against δN_c ; since $\delta N_c = (1 - \epsilon/3)\zeta_{\text{Mald}}$ (Eq. (5)) at $\epsilon = 3/2$ gives $1 - \epsilon/3 = 1/2$, an f_{NL} normalized against δN_c instead would differ from the values quoted here by a factor $1/(1 - \epsilon/3) = 2$ at $\epsilon = 3/2$ – numerically the same factor of two as Cai *et al.*'s slip (Sec. II), but a different mechanism: this one is a normalization-convention choice between ζ_{Mald} and δN_c , not an amplitude-conversion error, and the coincidence is not pursued further here. Neither equals the in-in result above. An anisotropic (Bianchi-I) separate-universe treatment does not close the gap either: its projected shear response is traceless and supplies no monopole (Appendix A, Sec. A 4).

The resolution is that δN on the zero-shift threading computes a different variable from Maldacena's ζ . On a uniform- ϕ slice the comoving-gauge shift is $N_i = \partial_i \psi$ with $\psi \supset a^2 \epsilon \partial^{-2} \zeta$, which is $\mathcal{O}(1/k_L)$ in a non-attractor phase – the gradient-expansion assumption $N_i = \mathcal{O}(\nabla)$ that underlies every separate-universe construction fails here. Separating the long mode's coupling into its local response ([L]: via $\zeta_L, \dot{\zeta}_L$), its shear/curvature ([K]: via $\partial_i \partial_j \tilde{\chi}_L$, which cancels exactly between the varied legs of the quartic vertex), its *shift* ([X]: via $\partial_i \tilde{\chi}_L \propto 1/k_L$), and short-mode sourcing ([S], which is suppressed), the classical $\mathcal{O}(k^0)$ super-Hubble solution reproduces the in-in result of Eq. (4) per vertex and for the full shape function: [X] alone carries 100% of the quadrupole and $+5/4$ of the monopole, with the $1/k_L$ poles cancelling only in the sum over vertices – poles no separate-universe construction (isotropic or Bianchi-I) contains. The zero-

shift-threading variable and ζ_{Mald} are related at linear order by

$$\delta N_c = \left(1 - \frac{\epsilon}{3}\right) \zeta_{\text{Mald}} \left(= \frac{1}{2} \zeta_{\text{Mald}} \text{ at } \epsilon = \frac{3}{2} \right), \quad (5)$$

derived from the Friedmann equation at fixed ϕ ($\delta H/H = -(\epsilon/3)\dot{\zeta}_L/H$) together with $\delta N_c = \int (K/3) d\tau$; at second order the map is a time-dependent, non-local (∂^{-2}) spatial diffeomorphism, so no local f_{NL} relation between δN_c and ζ_{Mald} exists, and comparing their squeezed limits directly (as the naive cross-check does) compares two different observables. The quadrupole gap is likewise closed by a computed identity, not merely bounded: the shift's dynamical coupling reproduces the anisotropic-rescaling ($\propto \mu^2$) piece exactly, and a final-time re-threading boundary term – absent from any separate-universe treatment – corrects it to the in-in coefficient exactly.

Scope statement.—The from-scratch in-in computation of Table I, independently fixing commutator, Wick-contraction, and orbit multiplicities from scratch and agreeing with two independently published results (Li *et al.*'s $c_s = 1$ limit and Quintin *et al.*'s quoted value), is the independent second-method route that adjudicates the factor of two *within the in-in method*, and it reproduces $-35/16$ exactly. The classical $\mathcal{O}(k^0)$ super-Hubble solution, organized by the shift decomposition above, provides a method-independent confirmation: it reproduces Eq. (4) exactly by a route independent of the in-in commutator expansion. The residual gap to every separate-universe δN value is explained at the equation level rather than reconciled away: at linear order the two calculations are related by the derived identity of Eq. (5); at second order the zero-shift-threading variable δN_c and ζ_{Mald} are related by a time-dependent, non-local spatial diffeomorphism whose net second-order effect on the squeezed bispectrum Appendix A records as an identity, without deriving the mechanism that produces it – so the naive squeezed-limit comparison of the two calculations' raw values compares two different variables, not two disagreeing predictions of the same one. This equation-level closure is established for the comoving-slice δN value (-5) and, following an independent adjudication ([research/theory_audit/a2_lapse_monopole_adjudication_2026_09_07.md](#)), for the uniform-density value as well: a direct second-order continuation of the uniform-density variable itself has squeezed monopole $f_{\text{NL}}^\rho = 5(\epsilon - 7)/8 = -55/16$ at dust, reproducing $-55/16$ quoted above exactly. The threading map's long $\times$ short lapse monopole coefficient $A_2 = \epsilon(3 - \epsilon)^2/3$ is the correct constraint-solve value (not $2(3 - \epsilon)^2$); the earlier initial-label figure $5(2\epsilon - 15)/24 = -5/2$ came from composing the map with the linear-mode weight $\lambda' = 2\lambda$, whereas the ρ -surface time shift, acting on the second-order curvature perturbation, carries weight 3λ , and correcting the weight closes the $15/16$ gap exactly. The in-in monopole $-15/8$ (dust, δN_c normalization), the comoving-slice δN value -5 , and the uniform-density δN value $-55/16$ are therefore three

well-defined variables related by exact threading maps, not competing claims about one number. The flagship in-monopole $-35/16$ (Maldacena, ζ_{Mald} normalization) is unaffected. We adopt Eq. (1) throughout this paper as the value of ζ_{Mald} , the variable every downstream channel (transmission, PTA, PBH, LSS) inherits, and carry $-35/8$ in parallel wherever a channel distinguishes them. Full derivations, the general- ϵ formula, and the $[L]/[K]/[X]/[S]$ decomposition are given in Appendix A and archived per the reproducibility statement below.

III. TRANSMISSION THROUGH THE BOUNCE

Equation (4) is a statement about the contraction phase alone, and it holds at every instant of that phase: Sec. II evaluates it in the limit $|k\eta_B| \rightarrow 0$ and its derivation is end-time independent at leading order (corrections are $\mathcal{O}(k^2\eta_B^2)$). Whether it survives into an observable post-bounce spectrum is a separate, narrower question that depends on physics not computed in full here, and we state the current evidence precisely under one explicit modeling assumption. **(A4)**: the contraction-phase bispectrum is frozen at a handoff time η_h near the NEC boundary and evolved purely linearly thereafter – i.e. we bound the mode-mixing *transfer* of the already-fixed contraction-phase amplitude, not the amplitude itself, which Sec. II has already shown is independent of when in the contraction phase it is evaluated. For a linear mode-mixing matrix that carries the growing mode’s amplitude through the bounce with weight 1 and the sub-dominant decaying mode with weight r_{mix} , writing $\rho(\eta_h) \equiv |J(-\eta_h)|/I_\infty \in (0, 1]$ for the fraction of the mode-mixing integral accumulated before η_h , the transmitted f_{NL} amplitude scales as $T_{f_{\text{NL}}}(\eta_h) = \frac{1 + r_{\text{mix}} [1 - \rho(\eta_h)]}{1 + 2r_{\text{mix}}}$, where $\mathcal{A}$ is the matter-asymptotic normalization of the scale factor ($a \rightarrow \mathcal{A}\tau^2$), $I_\infty \equiv J(+\infty)$ is the super-Hubble mode-mixing integral of the growing-mode basis, and $r_{\text{mix}} = -9i\mathcal{A}^2 I_\infty/k^3 = \beta_- I_\infty/\alpha_-$ is in general complex, with α_-, β_- the matter-basis projection coefficients and the denominator $1 + 2r_{\text{mix}}$ fixing the vacuum normalization $C_1 = \alpha_- (1 + r_{\text{mix}})$, $C_2 = \beta_-$; over the validated range $k\eta_B \lesssim 10^{-2}$ used throughout this paper, $|r_{\text{mix}}| \gg 1$ holds numerically for all three backgrounds below, and in this growing-mode-dominated limit the transfer reduces to

$$T_{f_{\text{NL}}} = \frac{1 - \rho(\eta_h)}{2}, \quad (6)$$

so that, under assumption (A4) and within scheme S1, $0 \leq T_{f_{\text{NL}}} < 1/2$ since $\rho \in (0, 1]$: within this handoff scheme, linear transfer can only suppress the amplitude, never invert or amplify its sign; this bound is specific to scheme S1 – S2’s raw-ADM continuation (Sec. III A) lies outside assumption (A4) and transmits with $T_{f_{\text{NL}}} \approx 1.03$ (from $|\lambda_\zeta| = 0.97$). $T_{f_{\text{NL}}}$ is precisely the f_{NL} transfer, $T_{f_{\text{NL}}} = 1/\lambda_\zeta$ with $\lambda_\zeta \equiv \zeta(+\infty)/\zeta(-\eta_h)$ the linear-

amplitude growth of ζ across the handoff ($\lambda_\zeta = 4.0, 5.12, 6.06$ for the LQC, poly, and Quintin-type backgrounds respectively): ζ *grows* through the bounce by λ_ζ , which is why f_{NL} is suppressed rather than enhanced by a uniform rescaling. Separately, the post-bounce curvature power spectrum remains scale-invariant, $\Delta^2 \propto k^3 |\alpha_{\text{post}}|^2$ flat to 1.2–4.2% across the validated k range in the $|r_{\text{mix}}| \gg 1$ limit used here, so $n_s = 0.9649$ in Sec. VI is not in tension with this section. Evaluated across three explicit non-singular bounce backgrounds – an LQC effective quasi-dust bounce (“LQC”), an analytic non-LQC polynomial bounce (“poly (analytic non-LQC)”, a different functional bounce profile from the LQC effective-dust form, not an LQC sub-case), and the Quintin *et al.* [3]-type parametrization (“Quintin-type”) – at the natural hand-off $\eta_h = \eta_B$ (the NEC boundary), $T_{f_{\text{NL}}}$ ranges over 0.165–0.409: a 1.64 $\times$ spread between two mode-function conventions (geometric vs. effective-fluid) on the *same* LQC background (0.409/0.250), and a 2.48 $\times$ spread across the full set of three backgrounds and two conventions (0.409/0.165; the combined S1 range below uses the S1 rows only). The upper value, 0.409, is the LQC background under the effective-fluid convention (scheme S2); it is *not* a fourth background, and, as stated below, S2 has no computable *cubic-order* post-bounce $f_{\text{NL}}^{\text{after}}$ on the LQC background (the linear S2 transfer on LQC exists, $T = 0.409$; S2 is carried through cubic order only on the Quintin-type background, Sec. III A), so this LQC/S2 value does not enter the combined S1 range that follows. This bound is a statement about scheme-dependent linear transfer, not a universal property of the physical post-bounce f_{NL} .

Direction of the $k\eta_B$ window.—The validity condition $k\eta_B \lesssim 10^{-2}$ used throughout this section is an *upper* bound on k , not a lower one: it is satisfied *most easily* at the largest observable scales (the CMB/LSS pivot, $k \sim 0.05 \text{ Mpc}^{-1}$) and *hardest* to satisfy at the smallest, PBH-relevant scales ($k \sim 10^{15} \text{--} 10^{16} \text{ Mpc}^{-1}$ for a 10^{15} g PBH), where it requires a correspondingly high bounce energy (Sec. V C). Consequently the scheme-S1 transfer of this section is satisfied most comfortably at the LSS/CMB pivot used by Channel III below, and the transmitted amplitude $f_{\text{NL}}^{\text{after}} = T_{f_{\text{NL}}} \cdot f_{\text{NL}}^{\text{before}} + \Delta f_{\text{NL}}^{\text{bounce}}$ (Sec. III A) is therefore this paper’s observable prediction for that channel, not the pre-bounce value.

What this bound does not include is behavior at $k\eta_B \gtrsim 10^{-2}$, outside the validated band, where three mechanisms have now been checked directly rather than left as a citation. **(1) Linear transfer at finite k .** Evaluated exactly (not in the super-Hubble limit) across $k\eta_B \in [0.1, 10]$ on all three backgrounds, the power-spectrum transfer relative to the S1 super-Hubble prediction is flat to 1–2% outside $k\eta_B \in [0.2, 2]$ but shows a real, background-dependent excursion, a factor 0.06–2.2 (not order-unity in the strict sense), peaking at $k\eta_B \approx 0.6$ –0.8: the power ratio reaches 1.76 (Quintin-type), 2.24 (LQC), and 0.058 (poly non-LQC). **(2) The Quintin *et al.* scalar-field-velocity-dip amplification,** their

Eq. (79): in their bounce-phase ansatz $H(t) = \Upsilon(t - t_B)$, $a(t) = a_B e^{\Upsilon(t - t_B)^2/2}$, $\Upsilon \equiv \dot{H}$ is the constant bounce-phase expansion rate and Δt_B the bounce duration in the NEC-violating window, related by $\Upsilon = 8/(3\Delta t_B^2)$; this program's grid point $\Delta t_B = 1$ gives $\Upsilon = 2.667$. Evaluated for this $(\Upsilon, \Delta t_B)$, the required amplification factor is exactly 1 on every background (constant on the Quintin-type background by construction, and not definable as a matter-sector effect on the two effective-fluid backgrounds), so no carrier for their mechanism exists here. **(3) The loop-quantum-cosmology cubic self-interaction enhancement** [7]: its third-order Hamiltonian is, by the authors' own statement, the Legendre transform of the classical cubic action of Ref. [5] and contains no operator dependent on ρ/ρ_c or the dressed potential; every one of its nine terms maps exactly onto scheme S1's vertex table; the operator is not absent from S1. Evaluated with the *exact* LQC-dust dressed-metric mode functions – not the super-Hubble reduction – across $k\eta_B \in [0.1, 10]$ and three choices of initial vacuum state, the resulting bispectrum shows no counterpart of the literature's order- 10^3 plateau and no feature at the literature's own enhancement scale $k_{\text{LQC}}\eta_B \approx 1.06$: at and below k_{LQC} (squeezed configuration, $k\eta_B \lesssim 1.06$) it sits 2.1–4.4 decades below that plateau, with a state-dependence under 13%; this deficit is not extended above k_{LQC} , where the lab/ABS ratio changes sign (Table II). None of the three mechanisms reopens the pulsar-timing or PBH nulls (Secs. IV D, V C). Any assumption that the bounce transmits f_{NL} with a coefficient near unity is therefore not supported by any of these calculations: the linear channel caps it at 1/2 within the (A4) handoff scheme.

TABLE II. Exact LQC-dust mode-function bispectrum $|f_{\text{NL}}^{\text{bounce}}|$ vs. the ABS literature plateau, lab-vacuum initial state (reproducibility statement below). The squeezed deficit is 2.1–4.4 dex only at and below $k_{\text{LQC}}\eta_B \approx 1.06$; above it the sign of the gap reverses.

configuration	$k\eta_B$	initial state	$ f_{\text{NL}}^{\text{bounce}} $
squeezed	0.1	lab vacuum	0.22
squeezed	0.3	lab vacuum	2.34
squeezed	1	lab vacuum	0.94
squeezed	3	lab vacuum	0.94
squeezed	10	lab vacuum	1.90
equilateral	0.1	lab vacuum	0.54
equilateral	0.3	lab vacuum	5.16
equilateral	1	lab vacuum	9.81
equilateral	3	lab vacuum	111.97
equilateral	10	lab vacuum	1215.57

A. The bounce's own cubic term: now computed

The intrinsic bounce contribution left open above is closed for one explicit scheme. In the geometric (dressed-metric) prescription $z = a$, $\epsilon_{\text{eff}} = 1/2$, $c_s = 1$ – scheme S1, the substitution of these values into the classical comoving-gauge $P(X, \phi)$ cubic action of Ref. [5], an assumption of the scheme and not a derivation of the quantum-gravity-dressed operator – the squeezed-isosceles bounce contribution evaluated over the window $[-\eta_B, \eta_B]$ is, with $\rho_B \equiv \rho(\eta_B) = 1 - 2T_{f_{\text{NL}}}$ the same handoff fraction of Sec. III evaluated at the bounce,

$$\Delta f_{\text{NL}}^{\text{bounce}}[\text{S1}] = -\frac{5}{24}\rho_B, \quad (7)$$

so that, combined with the linear transmission of the contraction-phase amplitude below, $f_{\text{NL}}^{\text{after}} = -\frac{85}{48}T_{f_{\text{NL}}} - \frac{5}{24}$, a closed-form vertex-table result confirmed to 3×10^{-4} by an independent finite- k numerical in-in evaluation of all six cubic attachments and boundary terms (Table I's five rows with the field-redefinition row's local and non-local parts counted separately, as the numerical pipeline treats them); the vertex $\zeta\zeta^2$ carries 99.95–99.97% of it. Combining Eq. (7) with the linear transmission of Eq. (1) at $\eta_h = \eta_B$, $f_{\text{NL}}^{\text{after}} = T_{f_{\text{NL}}} \cdot (-35/16) + \Delta f_{\text{NL}}^{\text{bounce}}$, gives a definite post-bounce prediction for the first time, one number per background: The LQC/S2

TABLE III. Per-background transmitted amplitude, scheme S1 (Sec. III); the S2 row is the Quintin-type-only raw-ADM continuation and is not directly comparable to the S1 rows above it (different scheme, no separately regulated $\Delta f_{\text{NL}}^{\text{bounce}}$).

background	$T_{f_{\text{NL}}}$	$\Delta f_{\text{NL}}^{\text{bounce}}$	$f_{\text{NL}}^{\text{after}}$
Quintin-type [3] (S1)	0.165	−0.140	−0.501
LQC effective dust (S1)	0.250	−0.104	−0.651
poly (analytic non-LQC) (S1)	0.196	−0.127	−0.555
Quintin-type (S2, raw-ADM)	≈ 1.03	–	−1.25

(effective-fluid) value $T_{f_{\text{NL}}} = 0.409$ has no row above: S2 has been carried through cubic order only on the Quintin-type background (below), so $|f_{\text{NL}}^{\text{after}}| \in [0.50, 0.65]$ (S1, all three backgrounds) against $|f_{\text{NL}}^{\text{before}}| = 2.19$: the bounce's own term is negative – it enhances $|f_{\text{NL}}|$, partially undoing but not reversing the linear suppression, at 19–39% of the transmitted contraction term. This range is valid only for $k\eta_B \lesssim 10^{-2}$ (set by the competing requirements that the post-bounce integral converge and the super-Hubble boundary treatment hold) and only within scheme S1.

Scheme S2, resolved.—The effective-fluid scheme S2's cubic bounce-window integral, previously reported divergent with no quoted number, is now shown to be finite: the divergence is an artefact of the standard integrated-by-parts (Maldacena/Chen) form of the cubic action, which introduces explicit $1/H$ factors singular exactly

at the bounce; on the exact linear S2 mode functions the comoving-gauge lapse and shift, $N_1 = \dot{\zeta}/H$ and $\psi = -\zeta/H + \chi$, are individually singular but their sum is regular at $H = 0$ (a residue cancellation), so the *raw* ADM cubic Lagrangian, taken before any integration by parts, is finite on-shell through the bounce and requires no cutoff. Evaluated on this raw form with the exact S2 modes on the Quintin-type background, the bounce-window in-in integral gives, at $k\eta_B = 10^{-3}, 3 \times 10^{-3}, 10^{-2}$ respectively, $f_{\text{NL}}^{\text{after}}[\text{S2}] = -1.249, -1.246, -1.244$ against $f_{\text{NL}}^{\text{before}} = -2.187 \approx -35/16$ – a factor ≈ 2.5 above S1’s -0.501 on the same background. The difference is dominated by the choice of linear Mukhanov–Sasaki variable, not by any single cubic operator: S2 transmits the growing mode almost unsuppressed ($|\lambda_\zeta| = 0.97$) where S1 amplifies it by 6.06, and the net bounce-plus-expansion cubic contribution is $+1.0$ (S2) against -0.14 (S1), traced by an operator-by-operator decomposition to a geometric-versus-lapse cancellation rather than a single dominant vertex. S1 and S2 are therefore genuinely physically inequivalent continuations through $H = 0$, not two regularizations of the same answer, and the paper’s own previously stated open item – whether the transmitted amplitude is scheme-independent – is answered negatively: at $f_{\text{NL}}^{\text{local}} = -35/16$ the transmitted amplitude is reported as the two-scheme band $f_{\text{NL}}^{\text{after}} \in [-1.25, -0.50]$ on the Quintin-type background (the S1 range across all three backgrounds is $[-0.65, -0.50]$; S2 is evaluated only on Quintin-type). Because the raw-form S2 bounce-window integral is not bounce-localised (an $O(1)$ dependence on the evaluation-window convention, unlike S1’s), only $f_{\text{NL}}^{\text{before}}$ and $f_{\text{NL}}^{\text{after}}$ are convention-free S2 quantities; no separately regulated $\Delta f_{\text{NL}}^{\text{bounce}}[\text{S2}]$ analogous to Eq. (7) is reported, and S2’s row for the Cai amplitude $-35/8$ is not computed (Table VI).

Quintin *et al.*’s reported bispectrum growth through a nonsingular bounce [3] is not contradicted: their growth is the *additive* bounce-generated term, the counterpart of Eq. (7), sharing its sign (both enhance $|f_{\text{NL}}|$); the factor-of-two suppression identified above is the *multiplicative* transmission of the inherited contraction amplitude, a quantity they do not compute, so the two statements are about different terms and are not in tension. The nonlinear channel is therefore no longer an unknown but a scheme-qualified prediction reported as a band; every use of $f_{\text{NL}}^{\text{after}}$ in this paper carries its scheme qualifier, and the unqualified statement that the bounce produces no orders-of-magnitude enhancement is addressed directly in Sec. III rather than left open. The full vertex table, regularization-assumption note, numerical cross-check, and literature-comparison derivation are archived and referenced in the reproducibility statement below.

IV. CHANNEL I: THE PULSAR-TIMING SPECTRAL SLOPE

A. Spectral conventions

With the characteristic strain parameterised as $h_c(f) \propto f^{(3-\gamma)/2}$ and $\Omega_{\text{GW}} = (2\pi^2/3H_0^2) f^2 h_c^2(f)$, the energy density scales as

$$\Omega_{\text{GW}}(f) \propto f^{5-\gamma}. \quad (8)$$

Hence $\gamma = 13/3$ for gravitational-wave-driven supermassive-black-hole binary inspirals ($\Omega_{\text{GW}} \propto f^{2/3}$), $\gamma = 5$ for a scale-invariant primordial tensor background ($n_{\text{T}} = 0$, $\Omega_{\text{GW}} \propto f^0$), and $\gamma = 2$ for $\Omega_{\text{GW}} \propto f^3$, the universal causality-limited infrared tail of an induced background sourced by a finite-duration process [9]. $\gamma = 3$ ($\Omega_{\text{GW}} \propto f^2$) is the slope Papanikolaou [8] derives for a matter bounce whose source spectrum carries a specific near-linear low- k growth, $P_{\mathcal{R}} \propto k$; we retain it below only as a contrast case, since §IV D shows this model’s own committed spectrum does not have that growth and does not reproduce $\gamma = 3$.

B. Official posterior and this program’s refit

The primary comparison for this channel is NANOGrav’s own published result: fitting the 15-yr Hellings–Downs-correlated free-spectrum posterior on the lowest 14 frequency bins with a power law, NANOGrav report $\gamma_{\text{HD}} = 3.2_{-0.6}^{+0.6}$, quoted explicitly as a posterior median and 5–95% interval [11]; a Gaussian-equivalent $1\sigma \approx 0.6/1.645 \approx 0.365$. Against this official posterior, $\gamma = 3$ (matter-bounce, $\Omega_{\text{GW}} \propto f^2$) sits at 0.55σ , $\gamma = 5$ (scale-invariant primordial tensors, $n_{\text{T}} = 0$) at 4.94σ , and $\gamma = 13/3$ (SMBHB) at 3.1σ – consistent with NANOGrav’s own characterization of $13/3$ as in “moderate tension ... at the 99% credible boundary.”

As a secondary, differently-conditioned cross-check, we independently refit the same Ceffyl kernel-density free-spectrum posteriors using *all* 30 frequency bins [11, 12] ($T_{\text{obs}} = 16.03$ yr) with a power law ($\gamma, \log_{10} A$) under priors $\gamma \sim U[0, 7]$, $\log_{10} A \sim U[-18, -11]$, using 32 walkers and 10^4 production steps after 2500 burn-in (3.2×10^5 samples; integrated autocorrelation time $\tau = (58.1, 58.0)$ for $(\gamma, \log_{10} A)$, giving $\text{ESS} = N/\max(\tau) = 5.5 \times 10^3$). The marginal is

$$\gamma = 2.567 \pm 0.382, \quad \log_{10} A = -14.03 \pm 0.38, \quad (9)$$

with median 2.591 and 68% interval $[2.304, 2.882]$; this refit pipeline is validated by injection (Sec. IV C) and sits 1.20σ from the official posterior (quadrature-combined $\sigma = 0.53$ from the two marginals’ widths). Table IV reports both the official-posterior tensions and Gaussian z -distances against the refit’s own marginal, plus Savage–Dickey density ratios against the refit, $B_{\gamma_*/\text{free}} = p(\gamma_*/d)/\pi(\gamma_*)$ with $\pi = 1/7$, quoted only where the chain

TABLE IV. Channel I. Tensions against the official NANOGrav 14-bin posterior ($\gamma_{\text{HD}} = 3.2^{+0.6}_{-0.6}$, $\sigma \approx 0.365$) and against this program’s 30-bin refit (Eq. (9)), with the refit’s Savage–Dickey factor where the chain has posterior support; the official posterior is the primary comparison and the refit is a secondary, differently-conditioned cross-check, not directly comparable to it. The causal floor ($\gamma = 2$, $\Omega_{\text{GW}} \propto f^3$, the universal IR bound discussed below in this section) is included as a comparator alongside the three physical hypotheses. All values reproduced from the committed chain (`pta_gamma_reproduce.py`); differences from a self-reproduction run of the same script against the same chain are $\leq 3 \times 10^{-15}$ (floating-point-level agreement, not an independent external check). z_{refit} at $\gamma = 5$ is a Gaussian approximation only: 0 of 3.2×10^5 chain samples reach $\gamma \geq 5$ (chain max 4.70), so no data-supported Bayes factor is reported there (see text). The “Prim. tensors” row’s $\gamma = 5$ is the scale-invariant reference value; this model’s own first-order tensor background instead predicts $\gamma_{\text{pred}}^{(1)} = 5.035$ (reproducibility statement below), within 0.04 of it and subject to the same null verdict below.

Hyp.	γ_*	slope	z_{off}	z_{refit}	$B_{\gamma_*/\text{free}}$
Causal floor	2	f^3	3.29σ	1.48σ	0.54
Matter bounce, borrowed $\gamma = 3$	3	f^2	0.55σ	1.14σ	3.23
SMBHB	13/3	$f^{2/3}$	3.1σ	4.63σ	5×10^{-4}
Prim. tensors	5	f^0	4.9σ	6.37σ	n/a

has posterior support: with zero of 3.2×10^5 samples at $\gamma \geq 5$ (chain maximum 4.70), no Bayes factor at $\gamma = 5$ is reported – a KDE evaluated there is extrapolating into an unsampled tail, and its value moves by an order of magnitude under a different bandwidth choice. The $\gamma = 13/3$ factor rests on 9 samples at $\gamma \geq 13/3$ and is quoted to one significant figure for the same reason.

The 1.14σ and 4.63σ refit entries are Gaussian approximations to a marginal that is not itself Gaussian (it is bounded by the sampling prior); directly from the chain, $P(\gamma > 3) = 8.97\%$. Narrowing the refit’s γ prior from $U[0, 7]$ to $U[2, 5]$ moves $B_{3/\text{free}}$ from 3.23 to 1.47 while leaving $B_{\text{MB/SMBHB}} = B_{3/\text{free}}/B_{13/3/\text{free}} \approx 7 \times 10^3$ ($\log_{10} B \approx +3.9 \pm 0.2$, one significant figure given the 9-sample tail estimate) stable in the range $7\text{--}9 \times 10^3$; the individual Savage–Dickey factors $B_{\gamma_*/\text{free}}$ against the refit’s own prior (“nested” since each γ_* is a point restriction within the refit’s free- γ model) are prior-sensitive, but their ratio $B_{\text{MB/SMBHB}}$ (the model ratio) is not.

C. Refit validation

The refit’s likelihood/prior construction (30 bins, $T_{\text{obs}} = 16.03\text{ yr}$, $\gamma \sim U[0, 7]$, $\log_{10} A \sim U[-18, -11]$) is validated by an injection-recovery test that reuses that construction verbatim, using the REAL NANOGrav 15-yr HD-correlated free-spectrum KDE density grids from the public Zenodo record [10.5281/zenodo.8060824](https://zenodo.org/record/8060824/files/30f_fs{hd}_ceffyl/*.npy) (`30f_fs{hd}_ceffyl/*.npy`, sha256-verified). Each

bin’s real observed KDE curve (shape/width/skew preserved exactly) is re-centered along the $\log_{10} \rho$ axis to the model prediction at a chosen injected ($\gamma_{\text{true}}, \log_{10} A_{\text{true}}$), since a real dataset’s true signal is unknown and a ground-truth injection test requires a known target; this is the strongest injection achievable from real data without simulating a full new PTA dataset. At $\gamma_{\text{true}} = 13/3$, five realizations (the full 30-bin recovery plus four bootstrap bin-resamples) recover $\gamma = 4.336$ on average with a mean pull of $+0.016 \pm 0.044\sigma$ (std 0.0993 over the 5 realizations, standard error of the mean); at a $\gamma_{\text{true}} = 3$ control, five realizations recover $\gamma = 3.005$ with a mean pull of $+0.033 \pm 0.054\sigma$ (std 0.1203; script, JSON, and data manifest recorded in the reproducibility statement below). Both are consistent with unbiased recovery: both mean pulls are well under 0.1σ , though the per-realization scatter (std $\approx 0.10\text{--}0.12\sigma$ over 5 realizations) is comparable to, not smaller than, that threshold, tightening a secondary synthetic-density cross-check that substitutes a synthetic per-bin Gaussian density for the real KDE grids: $\gamma_{\text{true}} = 13/3 \rightarrow$ mean pull -0.026σ ; $\gamma_{\text{true}} = 3 \rightarrow$ mean pull $+0.068\sigma$ (`pta_injection_30bin_2026-09-02.py`) – both runs agree the pipeline is unbiased at well under 0.1σ .

D. The model’s own nHz prediction: a null

The official posterior and the refit above are the *measured* slope values; they are not by themselves a test of the matter bounce, since $\gamma = 3$ was not derived from this model’s own spectrum but borrowed from Papanikolaou [8]’s low- k tail, whose source carries a small-scale enhancement that §VC shows this model’s spectrum does not have. We therefore propagate this model’s own committed curvature spectrum, $\Delta_{\zeta}^2(k) = A_s(k/k_*)^{n_s-1}$ with $A_s = 2.1 \times 10^{-9}$, $k_* = 0.05\text{ Mpc}^{-1}$, $n_s = 0.9649$ (`outputs/inlab_delta2.zeta.2026-09-03.json`), through the scale-independent A2 bounce transfer and the standard radiation-era induced-GW kernel [10] (kernel validated against the published scale-invariant closed form, 0.822542 vs. 0.8222, before any lab spectrum was inserted). Every committed background at or above §VC’s $T_B \gtrsim 6 \times 10^9\text{--}6 \times 10^{10}\text{ GeV}$ puts the entire NANOGrav band at $k\eta_B \leq 2.3 \times 10^{-8}$, six or more orders inside the transfer’s validity domain, so this result is not limited by transfer uncertainty.

The fitted slope over $f \in [2, 60]\text{ nHz}$ is

$$\begin{aligned} \gamma_{\text{pred}} &= 5.070 \quad (n_s = 0.9649), \\ \gamma_{\text{pred}} &= 5.000 \quad (n_s = 1 \text{ dust bracket}), \end{aligned} \quad (10)$$

reproducing the analytic self-similarity result $\gamma = 5 - 2(n_s - 1)$ to 1 part in 10^4 , with predicted amplitude $\Omega_{\text{GW}} h^2(f_{\text{yr}}) = 1.45 \times 10^{-23}$ (5.88×10^{-23} for the dust bracket) against NANOGrav’s power-law value $\Omega_{\text{GW}} h^2(f_{\text{yr}}) = 2.622 \times 10^{-8}$ at f_{yr} , using the free- γ posterior amplitude ($\log_{10} A \approx -14.19$, $A \approx 6.46 \times 10^{-15}$)

paired consistently with the free- γ slope $\gamma_{\text{HD}} = 3.2$ compared against below (source file in the reproducibility statement below). Two independent failures against the borrowed $\gamma = 3$: the slope is a Gaussian-equivalent 5.1σ (4.9σ , dust bracket) z -distance from NANOGrav’s measured $\gamma_{\text{HD}} = 3.2 \pm 0.6$ (a 5–95% interval width, consistent with the Savage–Dickey stance of Sec. IV), and the amplitude is a shortfall of $10^{15.2}$ ($10^{14.6}$, dust bracket) – fifteen orders of magnitude below the PTA band. The model predicts an induced-GW background far below current sensitivity and does not account for the NANOGrav signal: this channel is a **null**, not a supporting consistency.

The comparison above is the *second-order* (scalar-induced) background only; the model’s own *first-order* (primordial) tensor background at nHz, propagated with the same $f \leftrightarrow k$ map and transfer prefactor and the same background’s tensor tilt $n_T = n_s - 1$, dominates the induced background by roughly six decades at the CMB bound $r < 0.036$ [24] ($\Omega_{\text{GW}}^{(1)} h^2(f_{\text{yr}}) < 2.5 \times 10^{-17}$ against the induced 1.45×10^{-23}), and by 9.1 decades using the model’s own $r_{\text{after}} = 24$ first-order tensor background of Sec. VII ($\Omega_{\text{GW}}^{(1)} h^2(f_{\text{yr}}) = 1.7 \times 10^{-14}$, propagated with the same kernel), which is already excluded at k_* by the same BICEP/Keck bound independently of any of the three channels. It remains 8–9 orders of magnitude below NANOGrav’s 2.622×10^{-8} , and its slope, $\gamma_{\text{pred}}^{(1)} = 5.035$, is within 0.04 of the induced value, so Channel I is a null in both amplitude and slope whichever tensor order is compared (reproducibility statement below).

Reaching $\gamma = 3$ would require $P_{\mathcal{R}} \propto k^1$ sustained across the full 2–60 nHz band (the lab spectrum instead has $n_s - 1 = -0.0351$, or 0 in the dust bracket) and a $10^{7.2}$ amplitude enhancement at $k \approx 10^7 \text{ Mpc}^{-1}$ – precisely the PBH-forming regime §VC shows this model’s spectrum falls seven decades short of. Alternatively, placing the required feature at the bounce scale itself needs $T_B \approx 2.3 \text{ GeV}$, 9.6 decades below §VB’s own $T_B \gtrsim 6 \times 10^9 - 6 \times 10^{10} \text{ GeV}$ condition. Channels I and II therefore assume mutually inconsistent spectra if $\gamma = 3$ is claimed, and the lab’s own spectrum sides with Channel II’s null. We retain the $\gamma = 3$ citation only as a flagged contrast case, requiring an enhancement this model does not produce, not as this model’s prediction. Nor does this channel constrain f_{NL} even under that contrast case: it measures a slope, not a bispectrum amplitude, and would be identical at $-35/16$ and $-35/8$.

V. CHANNEL II: PRIMORDIAL BLACK HOLE ABUNDANCE

A. Press–Schechter with local quadratic non-Gaussianity: a truncated-map bound

Writing $\zeta = \zeta_G + A(\zeta_G^2 - \sigma^2)$ with $A = \frac{3}{5} f_{\text{NL}}$ and $\zeta_G \sim \mathcal{N}(0, \sigma^2)$ [13, 14], the mass fraction is $\beta(\zeta_c) = P(\zeta > \zeta_c)$, obtained exactly by inverting the quadratic. For $f_{\text{NL}} < 0$

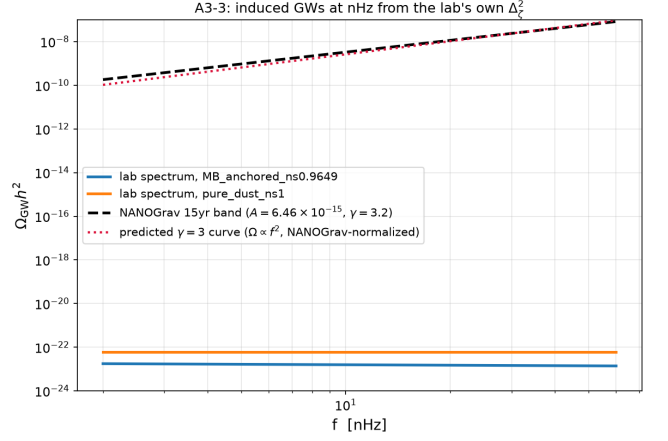


FIG. 1. Channel I null: $\Omega_{\text{GW}} h^2(f)$ induced by this model’s own committed curvature spectrum (CMB-anchored $n_s = 0.9649$ and the $n_s = 1$ dust bracket), propagated through the validated Kohri–Terada radiation-era kernel [10] and the scale-independent A2 transfer, against NANOGrav’s 15-yr power-law posterior ($\gamma_{\text{HD}} = 3.2 \pm 0.6$). The predicted slope, $\gamma_{\text{pred}} = 5.07$ (5.00, dust), is a Gaussian-equivalent 5.1σ (4.9σ) z -distance from the measured slope; the predicted amplitude is $10^{15.2}$ below the NANOGrav band. Source: `outputs/sigw_nhz_from_lab_spectrum_2026-09-04.json`.

this map is a downward parabola with an absolute ceiling

$$\zeta_{\text{max}} = -\frac{5}{12 f_{\text{NL}}} + \frac{3}{5} |f_{\text{NL}}| \sigma^2, \quad (11)$$

whose *leading* term scales as $1/|f_{\text{NL}}|$ and exactly doubles when $|f_{\text{NL}}|$ is halved; quoting that leading term alone gives $\zeta_{\text{max}} = 0.09524$ at $-35/8$ and 0.19048 at $-35/16$. The subleading $\frac{3}{5} |f_{\text{NL}}| \sigma^2$ term is dropped from those two numbers; restoring it at a representative $\sigma = 0.1$ gives the full $\zeta_{\text{max}} = 0.1215$ at $-35/8$ and 0.2036 at $-35/16$ (ratio 1.68, not exactly 2). At the standard curvature thresholds $\zeta_c \approx 0.45$ –1 this ceiling forces $\beta \equiv 0$ identically for *both* candidate values, so a first pass restricted to the low-threshold regime ($\zeta_c \lesssim 0.15$) is the only place the quadratic map produces a nonzero answer at all – and there it reported $f_{\text{PBH}}(-35/16) > f_{\text{PBH}}(-35/8)$ by three to ten orders of magnitude, with the halved value landing inside the $10^{-3} \leq f_{\text{PBH}} \leq 1$ band reported by Choudhury *et al.* [16] for negative- f_{NL} bounce scenarios. That ordering was the sharpest statement the channel could make, but the ceiling that produced it is an artefact of truncating the local map at quadratic order, not a physical statement: it is exactly why the literature this comparison is drawn against uses the compaction-function formation criterion instead [16], and redoing the calculation on that criterion is the subject of the rest of this section.

B. The compaction-function result

Following Young, Byrnes & Sasaki (2019) [17] and Choudhury *et al.* (2025) [16], formation is set by the peak compaction $C(r_p) = \delta_p$ exceeding a threshold C_{th} , with the radiation-era compaction function $C = C_{\text{lin}} - C_{\text{lin}}^2/(4f(w))$, $f(w) = 2/3$, and $C_{\text{lin}} = JC_G$ carrying the same quadratic local map ($J = 1 + \frac{6}{5}f_{\text{NL}}\zeta_G$) applied to the linear compaction C_G rather than to ζ itself. Because C_G is an unbounded Gaussian variable, the map $\zeta_G \rightarrow C_{\text{lin}}$ has *no* ceiling, so every threshold $C_{\text{th}} < C_{\text{max}} = f(w) = 2/3$ is reachable at every f_{NL} : the Eq. (11) artefact is removed by construction. We integrate the joint Gaussian PDF of (C_G, ζ_G) over the type-I formation domain (Musco 2019 [18]; C_{th} scanned over $\{0.4, 0.5, 0.6\}$) with a critical-scaling mass function ($\gamma_{\text{crit}} = 0.36$, $K = 4$), using the Gaussian window for C_G and the spherical window (with radiation transfer function) for ζ_G , exactly as in Choudhury *et al.* Eqs. (30–66). The one ingredient of their calculation that is not reconstructible from the published paper is the regularized-renormalized- resummed one-loop curvature spectrum sourced by their EFT-of-bounce-plus-USR mode history: no closed-form $\Delta_\zeta^2(k)$, loop-counterterm normalisation, or EFT parameter set is printed. We use a lognormal stand-in spectrum, scanned over width $\Delta \in \{0.35, 0.5, 0.8\}$ and peak-scale $r_p k_p \in \{0.75, 1.0, 1.5\}$, and scan the amplitude A rather than adopting theirs.

At fixed curvature amplitude, calibrated so the Gaussian case gives $f_{\text{PBH}} = 1$, this criterion *reverses* the first-pass ordering: across the full 27-point $(\Delta, r_p k_p, C_{\text{th}})$ grid, $f_{\text{PBH}}(-35/16) < f_{\text{PBH}}(-35/8)$ at *every* point (Table V, Fig. 2), because the correlation $\gamma_{\text{cr}} \equiv \sigma_{\text{cr}}^2/(\sigma_c \sigma_r)$ – with σ_c^2 the compaction-function variance, σ_r^2 the curvature variance, and σ_{cr}^2 their cross-variance, all evaluated with the same real-space top-hat window at the peak scale r_p used for C_{th} – that governs the sign of the non-Gaussian correction is large and positive (0.766–0.968) at these spectrum shapes: the Gaussian-case formation channel rides a correlated ridge that negative f_{NL} destroys, forcing the system into an anti-correlated, probability-costly regime whose leverage *grows* with $|f_{\text{NL}}|$. $-35/8$ therefore always out-produces $-35/16$ here – the opposite of the truncated-quadratic-map result. The one analytic statement of the first pass, that Eq. (11)’s leading term exactly doubles when $|f_{\text{NL}}|$ is halved (the full expression, including the dropped σ^2 term, does not), remains arithmetically true of that leading term but is now physically empty: it described a truncation artefact that the compaction criterion does not share.

f_{PBH} itself is not a quotable constraint: because it depends exponentially on γ_{cr} , it moves by more than 100 decades across the grid at fixed target amplitude (Table V), tracking the unreproducible spectrum shape (the primordial spectrum’s unreconstructible detailed profile, discussed further in Sec. VB). What *is* robust is the ratio of curvature amplitudes needed to reach a fixed target abundance, since K , Ω_{DM} , g_* , and M_H all cancel

in it: reaching the floor of the Choudhury *et al.* band, $f_{\text{PBH}} = 10^{-3}$, requires

$$\frac{A(-35/16)}{A(-35/8)} = 1.84 [1.76, 1.89], \quad \text{std} = 0.03, \quad n = 144, \quad (12)$$

restricted to $\gamma_{\text{cr}} \in [0.267, 0.630]$, the interval this model’s own near-scale-invariant spectrum shape occupies (on the narrower, original 27-point committed grid, $\gamma_{\text{cr}} \in [0.766, 0.968]$, the same ratio is 1.732 [1.610, 1.809], $\text{std} = 0.050$, and must not be quoted as universal), i.e. stable to $\pm 2\%$ – 6% while f_{PBH} itself moves by more than 100 dex. This ratio is the channel’s most reproducible output – it holds at every one of the 144 grid points in this model’s own γ_{cr} range and is robust to spectrum shape ($\text{std} = 0.03$ on a mean of 1.84; the original, narrower 27-point committed grid gives 1.732 ± 0.050 and must not be quoted as universal) – and it is a result within a stated regime, not a claim of general validity. *Regime of validity.* The result lives on the anti-correlated ($\gamma_{\text{cr}} > 0$, $J > 1$) branch identified above, and it holds at curvature excursions where the quadratic local map is not always perturbatively controlled: $1.2|f_{\text{NL}}|\sigma_r \approx 0.54$ – 1.01 at $-35/16$ and ≈ 1.09 – 2.02 at $-35/8$ across the grid – the same class of limitation Choudhury *et al.* report as their own $|f_{\text{NL}}| \lesssim 60$ perturbativity bound, and a limitation of the shared quadratic ansatz rather than of either calculation specifically. *Non-monotonicity.* At fixed curvature amplitude, $f_{\text{PBH}}(f_{\text{NL}})$ is strongly non-monotonic along the continuity path from the Gaussian case ($f_{\text{NL}} = 0$) to $-35/8$: it falls by ~ 55 decades to a minimum near $f_{\text{NL}} \approx -0.35$, then climbs back by ~ 53 decades through $-35/16$ to $-35/8$. Both candidate values lie past that minimum, on the branch where the anti-correlated ($\zeta_G < 0$) region of (C_G, ζ_G) space dominates formation and drives the ratio’s leverage; a reader inspecting only the two candidate points would not see this structure, so we record it here for transparency even though it does not change Eq. (12). The Gaussian-calibrated normalization amplitude at the representative grid point $(\Delta, r_p k_p, C_{\text{th}}) = (0.5, 1.0, 0.5)$ is $A_* = 0.131446$ (the amplitude at which $f_{\text{PBH}} = 1$ for $f_{\text{NL}} = 0$); Table V and Fig. 2 report f_{PBH} at this calibration. Reproducing Choudhury *et al.*’s reported band at $-35/8$ ($10^{-3} \leq f_{\text{PBH}} \leq 1$) is not forced by construction – the amplitude was fixed by the unrelated Gaussian-case normalisation – and it lands there for every γ_{cr} once the sign disagreement below $\gamma_{\text{cr}} \approx 0.85$ is traced to its source and resolved: the apparent *enhancement* there is an IR-divergent $O(\epsilon^2)$ artefact of the quadratic compaction map ($\propto (6\gamma_{\text{cr}}^2 - 1)\sigma_r^2$, positive only below $\gamma_{\text{cr}} = 1/\sqrt{6} \approx 0.408$ and formally divergent as the IR cutoff is removed), not a physical effect; the IR-finite $O(\epsilon)$ term, $\epsilon = (6/5)f_{\text{NL}}\sigma_r$, suppresses at first order in ϵ at every γ_{cr} , so negative f_{NL} suppresses the compaction abundance at first order, and Choudhury *et al.* are right.

TABLE V. Channel II. Compaction-function criterion: amplitude ratio $A(-35/16)/A(-35/8)$ required to reach $f_{\text{PBH}} = 10^{-3}$, and the *nominal (uncapped)* f_{PBH} columns – i.e. evaluated at the Gaussian-calibrated amplitude A_* with no upper cutoff applied – for representative points of the $(\Delta, r_p k_p, C_{\text{th}})$ grid (full 27-point grid in `outputs/pbh_compaction_fnl.json`). An extended 255-point scan over $\gamma_{\text{cr}} \in [0.196, 0.968]$ (two spectrum-shape families) gives 1.83 ± 0.05 overall (range $[1.61, 1.91]$); restricted to $\gamma_{\text{cr}} \in [0.267, 0.630]$, the interval this model’s own near-scale-invariant spectrum shape occupies, the ratio is 1.84 ± 0.03 (144 points, range $[1.76, 1.89]$); on the original 27-point committed grid ($\gamma_{\text{cr}} \in [0.766, 0.968]$) it is 1.732 ± 0.050 (range $[1.610, 1.809]$) and must not be quoted as universal. The ratio decreases monotonically, with a slope of ~ 0.13 per unit γ_{cr} across the 255-point scan, so the abstract’s 1.84 ± 0.03 is a shape-resolved scan result, not a union of a scan and an out-of-coverage point. The $\gamma_{\text{cr}} \lesssim 0.8$ rows below (*non-perturbative branch*; $1.2|f_{\text{NL}}|_{\sigma_r} \approx 0.5\text{--}2$, i.e. $O(\epsilon) \sim 1$ where the linearized criterion above is not controlled, IR-cutoff-dependent per the text) report $f_{\text{PBH}} > 1$ as *uncapped ratios* at the per-point Gaussian-calibrated A_* , not physical probabilities. The sign disagreement with Choudhury *et al.* is resolved above: negative f_{NL} suppresses the abundance, as they report (reproducibility statement below).

Δ	$r_p k_p$	C_{th}	γ_{cr}	$f_{\text{PBH}}(-35/16)$	$f_{\text{PBH}}(-35/8)$	ratio A
0.35	0.75	0.5	0.897	5.8×10^{-21}	4.5×10^{-7}	1.767
0.35	1.5	0.5	0.968	5.3×10^{-107}	7.0×10^{-59}	1.673
0.50	1.0	0.5	0.888	3.6×10^{-14}	1.6×10^{-2}	1.734
0.80	0.75	0.6	0.766 [†]	3.5×10^3	2.2×10^8	1.809
0.80	1.5	0.4	0.857	5.8×10^{-9}	1.6×10^1	1.677
mean ratio 1.732 [1.610, 1.809], std 0.050, $n = 27$						
[†] non-perturbative: $f_{\text{PBH}} > 1$ uncapped						

Present-day abundance uses [15]

$$f_{\text{PBH}} = \frac{1}{\Omega_{\text{DM}}} \left(\frac{M_{\odot}}{M_H} \right)^{1/2} \left(\frac{g_*}{106.75} \right)^{3/4} \left(\frac{g_{*s}}{106.75} \right)^{-1} \times \frac{\beta_{\text{NG}}}{7.9 \times 10^{-10}}, \quad (13)$$

with $\Omega_{\text{DM}} = 0.674$ ¹, $g_* = g_{*s} = 106.75$, and a single horizon mass $M_H = 10^{20}$ g (matching the first pass, rather

¹ Quoted as printed in Choudhury *et al.*’s Eq. (66); it numerically coincides with the Planck 2018 reduced Hubble parameter h , not the dark-matter density parameter ($\Omega_{\text{DM}} \approx 0.264$). The required-amplitude *ratio* (Eq. (12)) and this section’s null verdict are unaffected, since Ω_{DM} cancels identically there; the *tabulated* f_{PBH} column and Fig. 2 are calibrated so the Gaussian case gives $f_{\text{PBH}} = 1$ at this quoted value and would shift by the factor $0.674/0.264 \approx 2.55$ under the dark-matter-density value (since $f_{\text{PBH}} \propto 1/\Omega_{\text{DM}}$) – they are not independent of which Ω_{DM} is used, though this $O(1)$ rescaling changes none of this section’s conclusions: the model’s own predicted f_{PBH} is exactly zero in double precision regardless (Sec. VC), seven orders of magnitude below either normalization.

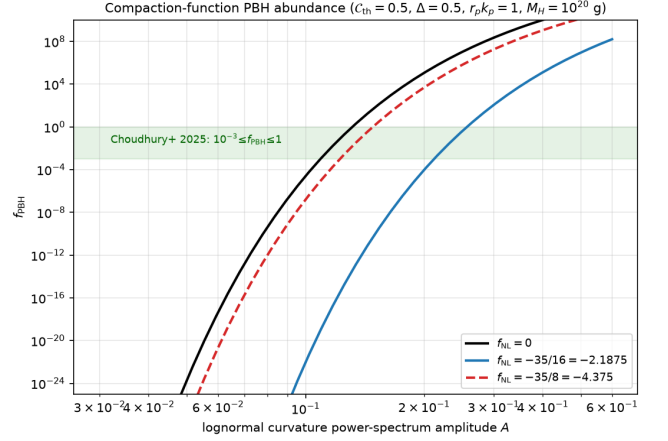


FIG. 2. Channel II, compaction-function criterion: f_{PBH} vs. curvature power-spectrum amplitude A at the representative grid point $(\Delta, r_p k_p, C_{\text{th}}) = (0.5, 1.0, 0.5)$, for the Gaussian case ($f_{\text{NL}} = 0$, calibrated to $f_{\text{PBH}} = 1$ at $A_* = 0.131446$) and the two candidate values; $-35/16$ requires higher A than $-35/8$ to reach the same f_{PBH} , from `pbh_compaction_fnl.py`. The continuity scan underlying the non-monotonicity discussed in the text is reported in `outputs/pbh_compaction_fnl.json`; the required amplitude-ratio grid (Eq. (12)) is in Table V.

than the mass-integrated Eq. 66 of Choudhury *et al.*, an $O(1)$ width-factor deviation for a narrow peak). Full formalism, the itemized deviations from Choudhury *et al.*’s setup (mass-integration, width-factor, and spectrum-shape approximations), and validation are recorded in the compaction note cited in the reproducibility statement below.

C. The lab’s own primordial spectrum: a null result

Sec. VB’s spectrum shape is a lognormal stand-in with a *scanned* amplitude, because Choudhury *et al.*’s spectrum is not reconstructible from their published paper. Removing that dependence, Eq. (12)’s amplitude ratio can instead be evaluated for the spectrum this matter-contraction model actually predicts: a near-scale-invariant power law, $n_s - 1 = 12w/(1 + 3w)$, anchored to the Planck 2018 amplitude $A_s = 2.1 \times 10^{-9}$ at $k_* = 0.05 \text{ Mpc}^{-1}$ and tilted by the small deviation from $w = 0$ ($n_s = 0.9649$), extrapolated across the 10–15 decades to PBH scales under two assumptions: the equation of state does not evolve away from its CMB-calibrated value, and the bounce transfer is scale-independent, which Sec. IIIA establishes only for $k\eta_B \lesssim 10^{-2}$: satisfied at every PBH mass scale considered here only if the bounce energy is high enough. Requiring $k\eta_B \lesssim 10^{-2}$ at the smallest PBH mass used here (10^{15} g, $k \sim 10^{15}\text{--}10^{16} \text{ Mpc}^{-1}$) needs $a_B H_B \gtrsim 10^{17}\text{--}10^{18} \text{ Mpc}^{-1}$, i.e. a bounce temperature $T_B \gtrsim 6 \times 10^9$

6×10^{10} GeV (using $k_B \simeq 1.71 \times 10^7 T_B [\text{GeV}] \text{Mpc}^{-1}$, `outputs/inlab_delta2_zeta_2026-09-03.json`) – thirteen decades above the BBN scale (~ 1 MeV); a scale-independent transfer with $T_{f_{\text{NL}}} < 1/2$ cannot by itself close the 7-decade amplitude shortfall found below, so this condition, while formally required, does not affect the null verdict. On this shape, inside the model’s own $\gamma_{\text{cr}} \in [0.267, 0.630]$ coverage the required-amplitude ratio is 1.84 ± 0.03 across a threshold scan $C_{\text{th}} \in \{0.4, 0.5, 0.6\}$: the ratio remains the channel’s robust, spectrum-shape-stable output, but the narrower 1.732 ± 0.050 from the original committed grid must not be quoted as if it were universal.

Fed into the same compaction machinery unmodified, the lab’s own spectrum delivers $\Delta_{\zeta}^2 \sim 5 \times 10^{-10} - 1.3 \times 10^{-9}$ at PBH scales against a required 0.006–0.019 (the compaction threshold): a shortfall of 6.7–7.0 orders of magnitude in curvature amplitude ($\log_{10}$ ratio 6.75 at $-35/8$, 7.02 at $-35/16$), at every mass from 10^{15} g to $10^4 M_{\odot}$, over the same threshold scan and a four-decade IR-cutoff scan. f_{PBH} is exactly zero in double precision at every point (Gaussian-limit $\log_{10} \beta \approx -1.7 \times 10^9$ at $M_H = 10^{20}$ g, i.e. an $n_{\sigma} = 89,149$ Gaussian excursion; $89149^2 / (2 \ln 10) = 1.73 \times 10^9$; `outputs/inlab_delta2_zeta_2026-09-03.json`). **The PBH channel is therefore a null for this model’s own predicted spectrum:** it neither constrains nor is evidenced by primordial black holes, and any quotable PBH signal would require a small-scale amplification mechanism the model does not contain – a survey of four such candidate mechanisms [3, 7] finds none applies within the model’s validated ($k\eta_B \lesssim 10^{-2}$) domain, and this is no longer only a literature survey: at $k\eta_B \approx 3$ (an illustrative bounce energy *below* §VB’s validity condition, $T_B = 10^8$ GeV, $k_B \approx 1.714 \times 10^{15} \text{Mpc}^{-1}$, so $k \approx 5.1 \times 10^{15} \text{Mpc}^{-1}$, inside this paper’s own PBH band), the extreme small-scale end of the PBH band, the scheme-S1 bispectrum has now been evaluated directly on exact (non-super-Hubble) LQC-dust mode functions (Sec. III) and gives $|f_{\text{NL}}| \approx 0.9\text{--}1.9$, which *worsens* rather than relieves the required Gaussian excursion (the required-excursion threshold, at the $M_H \sim 10^{15}\text{--}10^{16}$ g end where the 7.0-decade Δ_{ζ}^2 deficit above applies – a $10^{3.5} = 3162$ Gaussian- σ excursion – moves to 3364σ); even the largest number produced anywhere in that scan (equilateral configuration, $|f_{\text{NL}}| \approx 1.2 \times 10^3$) leaves the threshold at 408σ against the 7.0-decade amplitude deficit established here, so no configuration, initial state, or evaluation time tested reopens this channel.

The same spectrum is checked against the FIRAS μ -distortion bound ($|\mu| < 9 \times 10^{-5}$): the lab’s own prediction gives $\mu = 1.65 \times 10^{-8}$, safely allowed at 1.8×10^{-4} of the bound. For the early-supermassive-black-hole-seed channel (a possible use of PBH formation this ratio was originally motivated by), the *required* curvature amplitude for $10^3\text{--}10^6 M_{\odot}$ seeds is FIRAS-excluded by three orders of magnitude if broadband, and remains excluded for seeds $\geq 10^4 M_{\odot}$ even as a narrow ($\Delta = 0.5$) spectral

peak; only the lightest ($10^3 M_{\odot}$) seeds evade the bound with a sufficiently narrow peak. Combined with the null above, the honest statement is two-part: the required SMBH-seed amplitude is excluded by three orders of magnitude (broadband, and for $\geq 10^4 M_{\odot}$ narrow-peak seeds) independent of this model, and this model’s own spectrum falls far short of supplying it either way. Full derivation, the four-mechanism literature survey, and every number above are archived in the lab-spectrum script cited in the reproducibility statement below.

VI. CHANNEL III: THE LARGE-SCALE-STRUCTURE BISPECTRUM

A. Current constraints

The DESI DR1 analysis of 1,631,716 luminous red galaxies ($0.6 < z < 1.1$) and 1,189,129 quasars ($0.8 < z < 3.1$) [19] gives $f_{\text{NL}}^{\text{local}} = -3.6_{-9.1}^{+9.0}$ (asymmetric 9.0/9.1) at 68% confidence with a merger model for the quasar response, or $f_{\text{NL}}^{\text{local}} = +3.5_{-7.4}^{+10.7}$ (asymmetric 10.7/7.4) assuming universality. These two numbers come from mutually exclusive theoretical priors on the same DESI DR1 data (merger-response vs. universality) and are not directly comparable to each other; each is quoted here against its own prior. This section compares against the pre-bounce amplitude $f_{\text{NL}}^{\text{local}} = -35/16$, since that is the value the DESI DR1 analysis constrains directly, using the upper side of the merger-response error (+9.0, since the prediction lies above the central value) and the lower side of the universality error (−7.4, for the same reason); the transmitted, post-bounce range is compared against forecast reach below (Sec. VI, “Forecast reach”). The prediction sits 0.16σ from the former and 0.77σ from the latter. The discriminating power is $|f_{\text{NL}}|/\sigma = 0.24$: DESI DR1 is consistent with the prediction under either prior and cannot test it.

As an independent check of the pipeline these forecasts rest on, we re-analyze the public DESI DR1 QSO sample using DESI’s own published window matrix, full-randoms P_{ℓ} , and EZmock covariance ($z = 0.8\text{--}2.1$, $k \leq 0.08 h \text{Mpc}^{-1}$, 46 dof; b_1 and f_{NL} free, n_{shot} fixed at 0), and recover $f_{\text{NL}}^{\text{local}} = -2.2 \pm 25$ at $p = 1.6 - 0.06\sigma$ from DESI’s published $-3.6_{-9.1}^{+9.0}$, which reproduces the published constraint’s sign and scale while showing that DR1’s constraining power at this configuration is an order of magnitude too weak to separate $-35/16$ from $-35/8$, from zero, or from the published central value; the near-coincidence of the recovered central value with $-35/16$ is a coincidence, not evidence. This reproduction’s $\sigma = 25$ is $\sim 2.8\times$ the published 9.0 because wide-angle corrections (`PowerSpectrumOddWideAngleMatrix`) are not applied and only 2 of 5 imaging-systematics splits were run; full manifest cited in the reproducibility statement below.

TABLE VI. Channel III. Bare detection significance $|f_{\text{NL}}^{\text{after}}|/\sigma_{f_{\text{NL}}}$ of the transmitted amplitude across the three backgrounds of Sec. III (range shown; Table III gives per-background values). The pre-bounce row (assuming $T_{f_{\text{NL}}} = 1$) is a secondary reference, not this paper’s prediction. The scheme-S2 row is evaluated only on the Quintin-type background (Sec. III A, “Scheme S2, resolved”); no $-35/8$ value is computed for S2 (not scaled from $-35/16$; see text). Forecast σ values are quoted from the cited abstracts. MegaMapper is an unapproved design and is quoted only at its published order-unity level; no systematic budget is transferred to it. Every tabulated significance is an *upper bound*: it assumes a shape overlap $r_{\text{sh}} = 1$ with the local template, and the matter-bounce shape’s true overlap $r_{\text{sh}} < 1$ (not yet re-derived at this paper’s $f_{\text{NL}}^{\text{after}}$ fiducial) can only reduce these numbers. Values re-derived by `survey_reach_fnl.py` (`outputs/survey_reach_fnl.json`).

Survey	$\sigma_{f_{\text{NL}}}$	bare ($-35/16$)	bare ($-35/8$)
<i>scheme S1, all three backgrounds:</i>			
SPHEREx, bispectrum only [20]	0.7	0.7–0.9 σ	1.2–1.7 σ
SPHEREx, $P + B$ target [20, 21]	0.5	1.0–1.3 σ	1.7–2.4 σ
MegaMapper class [22, 23]	~ 1	0.50–0.65 σ	0.86–1.20 σ
<i>scheme S2, Quintin-type only ($f_{\text{NL}}^{\text{after}} \approx -1.24$ to -1.25):</i>			
SPHEREx, bispectrum only	0.7	1.78 σ	not computed
SPHEREx, $P + B$ target	0.5	2.49 σ	not computed
MegaMapper class	~ 1	1.25 σ	not computed
<i>pre-bounce secondary (assuming $T_{f_{\text{NL}}} = 1$):</i>			
SPHEREx, bispectrum only	0.7	3.13 σ	6.25 σ
SPHEREx, $P + B$ target	0.5	4.38 σ	8.75 σ
MegaMapper class	~ 1	2.19 σ	4.38 σ

B. Forecast reach

As established in Sec. III, the scheme-S1 $k\eta_B$ window is satisfied most easily at this channel’s CMB/LSS pivot scale, so the transmitted amplitude $f_{\text{NL}}^{\text{after}} = T_{f_{\text{NL}}} \cdot f_{\text{NL}}^{\text{pre}} + \Delta f_{\text{NL}}^{\text{bounce}} \in [-0.65, -0.50]$ ($-35/16$) or $[-1.20, -0.86]$ ($-35/8$) (Table III) is this paper’s observable prediction for Channel III, not the pre-bounce contraction-phase amplitude. Table VI reports the bare significance $f_{\text{NL}}^{\text{after}}/\sigma_{f_{\text{NL}}}$ across the three backgrounds of Sec. III, with the pre-bounce amplitude’s bare significance retained as a clearly-labelled secondary row (i.e. assuming $T_{f_{\text{NL}}} = 1$, the transfer-free case this paper does not otherwise support). The matter-bounce bispectrum is also not exactly the local shape; a noise-weighted shape overlap $r_{\text{sh}} < 1$, which has not been re-derived at the $f_{\text{NL}}^{\text{after}}$ fiducial used in this paper – so no numeral is quoted here – would let an experiment quoting σ_{local} constrain the bounce amplitude at $\sigma_{\text{local}}/r_{\text{sh}}$; the r_{sh} -projected numbers are omitted pending that re-derivation.

At the transmitted amplitude the two candidate values separate by $|\Delta f_{\text{NL}}^{\text{after}}| \approx 0.35\text{--}0.55$ against $\sigma_{f_{\text{NL}}} = 0.5\text{--}0.7$ at SPHEREx sensitivity: $0.5\text{--}1.1\sigma$, so this channel does not discriminate $-35/16$ from $-35/8$ at current forecast reach once scheme-S1 transmission is applied; it remains the only channel where the factor of two would be observationally consequential *if* the pre-bounce amplitude

survived transmission unsuppressed, which Sec. III shows it does not.

VII. THE TENSOR AMPLITUDE OF THE MODELLED BACKGROUND

Every result above is conditional on the canonical, constant-sound-speed dust contraction used for the amplitude and its transmission. That background is not free of its own tensor spectrum. In the dressed-metric scheme the tensor and scalar canonical variables obey the identical equation $\mu'' + (k^2 - a''/a)\mu = 0$ from the identical Bunch–Davies vacuum, so $r = 16\epsilon$ identically for this class of background, and the dust contraction’s $\epsilon = 3/2$ fixes $r = 24$. The tensor mode obeys $h'' + 2(a'/a)h' + k^2h = 0$ with $c_T = 1$: it contains no $1/H$, no ϵ , no c_s , and none of the scalar constraint variables, so the S1/S2 continuation ambiguity – a choice of scalar variable z through $H = 0$ – cannot act on it, and because the S1 scalar equation with $z = a$ is literally the tensor equation, the tensor transfer λ_T equals the S1 scalar transfer $\lambda_{\zeta}^{\text{S1}}$ *identically*, verified to 1.4×10^{-14} on the Quintin-type background at $k\eta_B = 10^{-3}, 3 \times 10^{-3}, 10^{-2}$ by an independent tensor-equation solver, and to 8.5×10^{-9} on the poly and LQC backgrounds (Sec. III). So $r_{\text{after}} = r_{\text{before}}(\lambda_T/\lambda_{\zeta})^2$ depends on the scheme only through the scalar denominator: $r_{\text{after}}^{\text{S1}} = 24.0$ is an identity, unchanged by the bounce, exceeding the BICEP/Keck bound $r < 0.036$ [24] by $\sim 670\times$; in S2 the scalar transfer under the raw-ADM continuation is $\lambda_{\zeta}^{\text{S2}} = 0.970$ against the shared tensor transfer $\lambda_T = 6.06$, so the bounce *amplifies* r by $(\lambda_T/\lambda_{\zeta}^{\text{S2}})^2 \approx 39\times$ to $r_{\text{after}}^{\text{S2}} \approx 9.4 \times 10^2$, $\sim 2.6 \times 10^4\times$ above the bound – the tensor problem is *worse*, not cured, in S2. On the Quintin-type background, the only one where the S2 scalar mode could be exactly constructed through $H = 0$ (S2 is not defined on the LQC/poly backgrounds’ $\dot{H} = 0$ crossings), both continuations exceed the bound by more than two orders of magnitude, so neither of the two schemes evaluated here rescues this background from the long-known matter-bounce large- r problem [2, 3], applied here to the lab’s own background rather than assumed from the literature. Similarly, the exact dust contraction gives $n_s = 1$; the observed $n_s = 0.9649$ is an anchor fixing the contraction’s equation of state to $w = -0.0029$ via $n_s - 1 = 12w/(1+3w)$, not a prediction of the model. The model’s one genuinely predictive tilt statement is the consistency relation $n_T = n_s - 1 = -0.035$, sharply different from single-field inflation’s $n_T = -r/8$, and a cheap, falsifiable discriminator that survives everything else in this section. Propagating this first-order tensor background to nHz through the same kernel used in Sec. IV D gives $\Omega_{\text{GW}} h^2(f_{\text{yr}}) = 1.7 \times 10^{-14}$, still $10^{6.2}$ below NANOGrav’s 2.622×10^{-8} (the same free- γ posterior amplitude used throughout Sec. IV D): the PTA null of Sec. IV D is unaffected by this section’s own tensor amplitude. A tensor-sense $r = 0.84$ is not used here:

it traces to the noise-weighted bispectrum shape-overlap coefficient r_{sh} of Sec. VI, a shape correlation and not a tensor-to-scalar ratio.

VIII. THE JOINT (r, f_{NL}) NO-GO AND THE CURVATON ROUTE

The textbook remedy for a large tensor-to-scalar ratio in a contracting background is a reduced constant scalar sound speed $c_s < 1$, which suppresses r while enhancing the scalar bispectrum. For the dust contraction with constant c_s , two independent symbolic routes (a general- ν Hankel limit and the exact $q = 2$ mode functions) give

$$r = 16\epsilon c_s^{2\nu-2} = 24 c_s, \quad (14)$$

reproducing Li, Quintin, Wang & Cai (2016) [4] Eq. (3.18) and row 10's $r = 24$ at $c_s = 1$. The squeezed local amplitude, from their Eq. (4.19), is

$$f_{\text{NL}}^{\text{pre}}(c_s) = -\frac{165}{16} + \frac{65}{8c_s^2}, \quad (15)$$

whose $c_s \rightarrow 1$ limit reproduces this paper's own from-scratch $-35/16$ exactly (Sec. II). The bounce's own cubic contribution, evaluated at the same constant c_s (scheme S1, $z = a$, $\epsilon_{\text{eff}} = 1/2$), generalizes Eq. (7) to

$$\Delta f_{\text{NL}}^{\text{bounce}}(c_s) = -\frac{5}{24}\rho_B \frac{6c_s^2 - 5}{c_s^4}, \quad (16)$$

reproduced by the full numerical in-in integral to 3×10^{-4} and reducing to Eq. (7) at $c_s = 1$ to 4×10^{-6} (reproducibility statement below); the c_s dependence enters the mode functions only through the sound horizon ($c_s^2 k^2$ in the Mukhanov-Sasaki equation, with the Bunch-Davies Wronskian already carrying $1/\sqrt{2c_s k}$), so the whole effect is carried by the vertex coefficients, dominated (99.97%) by $\zeta\dot{\zeta}^2$. Eq. (16) changes sign at $c_s = \sqrt{5/6} = 0.913$ and diverges as $1/c_s^4$, faster than the transmitted contraction term $Tf_{\text{NL}}^{\text{pre}} \propto 1/c_s^2$, so $f_{\text{NL}}^{\text{after}}(c_s) = Tf_{\text{NL}}^{\text{pre}}(c_s) + \Delta f_{\text{NL}}^{\text{bounce}}(c_s)$ is used below rather than the transfer term alone. This evaluation holds for the full $P(X, \phi)$ cubic action, including a nonzero P_{XXX} coefficient $\lambda \equiv X^2 P_{,XX} + (2/3)X^3 P_{,XXX}$: λ enters the quadratic action nowhere ($\partial c_s^2 / \partial P_{,XXX} = \partial \Sigma / \partial P_{,XXX} = 0$, so $r = 24c_s$ is exactly λ -independent), and its ζ^3 vertex in the bounce window is odd in conformal time about the symmetric bounce and cancels identically in the S1 in-in integral (verified to 2.8×10^{-7} across $\lambda/\Sigma \in [-1, 1]$, the matter-contraction line, and the DBI line; row19_lambda, reproducibility statement below), so Eq. (16) also holds for any λ . $f_{\text{NL}}^{\text{pre}}(c_s)$ itself changes sign at $c_s = 0.8876$ (negative above, positive below): on $c_s \in [0.444, 0.888)$, the branch that is otherwise CMB-viable by $|f_{\text{NL}}^{\text{after}}|$, the k -essence prediction is a *positive* local amplitude, so the flagship negative sign survives only for $c_s \gtrsim 0.89$. The transfer through the bounce is c_s -independent to $\lesssim 5 \times 10^{-11}$

TABLE VII. The joint (r, f_{NL}) window in c_s , with the bounce's own c_s -dependent cubic term (Eq. (16)) included at the same c_s (Quintin-type background, $T_{f_{\text{NL}}} = 0.165$; reproducibility statement below).

requirement	c_s	implied r
$r < 0.036$ (BICEP/Keck)	$< 1.5 \times 10^{-3}$	0.036
$f_{\text{NL}}^{\text{after}}$ at that c_s	$1.0\text{--}1.4 \times 10^{11}$	
$f_{\text{NL}}^{\text{after}} \in [-6.0, 4.2]$ (Planck[28] 68%)	≥ 0.624	≥ 15.0

across all three backgrounds, because lowering c_s only rescales the mode's effective wavenumber and the transfer is already flat over $k\eta_B \ll 1$.

Including the bounce's own term moves the Planck-viable boundary (evaluated against the correct 68% interval $[-6.0, +4.2]$, whose upper edge binds since $f_{\text{NL}}^{\text{after}} > 0$ throughout this window) from row 14's $c_s \geq 0.477$ ($r \geq 11.4$) to $c_s \geq 0.624/0.632/0.627$ (Quintin/LQC/poly, $r \geq 15.0/15.2/15.0$) – nearly background-independent because the bounce term $\propto \rho_B$ and the transmission $(1 - \rho_B)/2$ are anti-correlated – and, at the tensor-viable $c_s = 1.5 \times 10^{-3}$, raises $f_{\text{NL}}^{\text{after}}$ from row 14's $6\text{--}9 \times 10^5$ to $1.0\text{--}1.4 \times 10^{11}$: the boundary moves against the model at both ends. The two regions are now disjoint by a factor ~ 416 in c_s (BICEP/Keck-viable $c_s < 1.5 \times 10^{-3}$ against Planck-viable $c_s \geq 0.624$), up from ~ 314 when the bounce term is evaluated only at $c_s = 1$: no constant- c_s value keeps both the tensor ratio and the local non-Gaussianity within current bounds, and the no-go is strengthened, not relaxed, by evaluating both terms at the same c_s . Eq. (16)'s own sign flip at $c_s = 0.913$ sits close to $f_{\text{NL}}^{\text{pre}}$'s at $c_s = 0.888$ (Sec. III A), so both terms change sign within the same narrow window and the flagship negative sign survives only above it. This independently reproduces and strengthens Li *et al.*'s (2016) own no-go statement (their $r < 0.07$ era bound gives $c_s \lesssim 0.0029$; the tighter current $r < 0.036$ moves this to $c_s < 1.5 \times 10^{-3}$ and $f_{\text{NL}}^{\text{pre}} = 3.6 \times 10^6$, a factor 3.8 tighter). The alternative route of Quintin *et al.* (2015) [3] – a scalar-only super-Hubble amplification through the bounce rather than a sound-speed change – also fails on the three backgrounds used throughout this paper: their Eq. (31) requires an amplification $\lambda \gtrsim 49$ for $r < 0.12$, while the velocity-dip evaluation of Sec. III (Eq. 79 of that reference, evaluated on these backgrounds) finds $\lambda = 4.0\text{--}6.1$ with no net r suppression. This entire c_s -window analysis is carried out in scheme S1, where r and f_{NL} are computed on the same transmission; the alternative-route test of Quintin *et al.* above is likewise available only in S1. Scheme S2's tensor ratio is larger still ($r_{\text{after}}^{\text{S2}} \approx 9.4 \times 10^2$, Sec. VII), so the joint exclusion is not relieved – if anything it is strengthened – by moving to S2. Restoring the cubic coefficient λ does not open the window either: a scan over $\lambda/\Sigma \in [-1, 1]$, the matter-contraction line $\lambda/\Sigma = (1 - c_s^2)/(6c_s^2)$, and the DBI line $\lambda/\Sigma = (1 - c_s^2)/(2c_s^2)$ shifts $f_{\text{NL}}^{\text{pre}}$ only by the con-

stant $-30\lambda/\Sigma$ (a purely local-shaped operator, Li *et al.* Eq. 4.18) while leaving $r = 24c_s$ untouched (Sec. III A); the best case is the DBI line, giving $r_{\min} = 12.57$ at $c_s = 0.524$ ($349\times$ BICEP/Keck), and cancelling the $1/c_s^2$ growth in $f_{\text{NL}}^{\text{pre}}$ at the tensor-viable c_s would require $\lambda/\Sigma = 7/(16c_s^2)$, i.e. a sound-speed running $s = 39/16$, far outside the $|s| \ll 1$ regime under which the amplitude being cancelled is derived. *The single-field matter bounce, for the full $P(X)$ k -essence class, is therefore excluded jointly by r and f_{NL} on the backgrounds and channels used in this paper, independently of the cubic coefficient λ .* This sharpens, rather than contradicts, the paper’s earlier statements: the exact amplitude $-35/16$ and its transmitted values remain correct statements about this background, but that background is now shown to be observationally excluded on tensor grounds independent of f_{NL} itself, and no sound-speed fix rescues it.

A route that survives is a light curvaton-type spectator riding the same contraction. Because it obeys the same Mukhanov–Sasaki operator as the adiabatic mode at constant ϵ , it inherits the identical tilt $n_s - 1 = 12w/(1+3w)$, and for a quadratic curvaton with sudden decay fraction r_{dec} ,

$$r = \frac{24}{1 + \frac{4}{3}r_{\text{dec}}^2(M_{\text{pl}}/\sigma_*)^2}, \quad f_{\text{NL}}^{\text{curv}} = \frac{5}{4r_{\text{dec}}} - \frac{5}{3} - \frac{5r_{\text{dec}}}{6}, \quad (17)$$

(Lyth–Ungarelli–Wands [26]; Sasaki–Valiviita–Wands [27], applied here to Cai, Brandenberger & Zhang’s 2011 [25] matter-bounce curvaton), which is (r, n_s) -tunable to observational viability and gives an $O(1)$ non-Gaussianity in its own right ($+9.30$ to -1.25 for $r_{\text{dec}} \in [0.113, 1]$, negative for $r_{\text{dec}} > 0.581$). The contraction’s own intrinsic $-35/16$ does not disappear in this picture; it enters the observable bispectrum suppressed by the curvaton weight $(r/24)^2 \simeq 2 \times 10^{-6}$ at $r = 0.036$, below any planned survey’s reach for any tensor-viable $r < 11.5$ (< 23 after this paper’s own bounce transfer), and only becomes detectable for $r \gtrsim 23$ – essentially the single-field value this section excludes. Cai, Brandenberger & Zhang’s [25] own quoted $-320/\pi^4 \approx -3.29$ is arithmetic on their order-of-magnitude estimate, not an independently derived result, and is not adopted here. Converting this route into a concrete, checkable prediction requires an entropy-sector kinetic amplification mechanism and a demonstration that σ_* stays constant across the bounce backgrounds used in this paper; neither is modelled here (Sec. IX C).

IX. DISCUSSION

A. The joint statement

The matter-bounce prediction $f_{\text{NL}}^{\text{local}} = -35/16$ – independently re-derived here within the in-in formalism, confirming Li *et al.* (2016) and localizing the factor-of-

two slip in Cai *et al.* (2009), and now confirmed by a method-independent route (Sec. IID) – is a correct statement about a background that Secs. VII and VIII show is jointly excluded by its own tensor-to-scalar ratio and this same non-Gaussianity: no constant sound speed relieves both at once. Conditional on that background, the amplitude is not contradicted by any channel currently able to speak, but three of the four channels tested are honest nulls rather than supporting consistencies. Pulsar timing: propagating this model’s own committed curvature spectrum through the validated induced-GW kernel gives $\gamma_{\text{pred}} = 5.07$ (5.00, dust bracket), a Gaussian-equivalent 5.1σ (4.9σ) z -distance from NANOGrav’s measured $\gamma_{\text{HD}} = 3.2 \pm 0.6$, at an amplitude $10^{15.2}$ below the NANOGrav band (Sec. IV D); the $\gamma = 3$ slope quoted for a matter bounce elsewhere in the literature [8] requires a small-scale spectral enhancement this model does not have, and is retained only as a flagged contrast case. This channel is also insensitive to the value of f_{NL} and therefore does not test it either way. Primordial black holes yield a reproducible amplitude ratio (1.84 ± 0.03 within this model’s own γ_{cr} coverage) between the two candidate values, but the lab’s own predicted spectrum falls 6.7–7.0 orders of magnitude short of the required amplitude at every mass scale considered (Sec. V C): the channel is a clean null for this model, not an open promise. The same lab spectrum falls three orders of magnitude short of the amplitude an early-supermassive-black-hole-seed channel would require (Sec. V C): a third honest null. Transmission through the bounce is now a definite, scheme-qualified prediction rather than an open bound: within scheme S1 and $k\eta_B \lesssim 10^{-2}$, $f_{\text{NL}}^{\text{after}} \in [-0.65, -0.50]$ (Sec. III A). As Sec. III states, this $k\eta_B$ window is an upper bound on k and is satisfied *most easily*, not *hardest*, at the CMB/LSS pivot scale used by Channel III below, so this transmitted range is precisely the number that channel should be compared against. Galaxy surveys are the only channel that discriminates between the two candidate amplitudes at all, but at the transmitted amplitude that discrimination is weak: DESI DR1 is consistent with $f_{\text{NL}}^{\text{after}}$ at well under 1σ under either prior, and SPHEREx reaches only 0.7 – 0.9σ ($-35/16$) or 1.2 – 1.7σ ($-35/8$) bare significance (Table VI), and combining power spectrum and bispectrum widens the candidate separation to only 0.5 – 1.1σ at current or forecast LSS sensitivity, not the 3.13σ forecast detection significance quoted for the pre-bounce amplitude a naive reading of Channel III alone would suggest: the two figures are *not directly comparable*, since 3.13σ assumes $T_{f_{\text{NL}}} = 1$ (Table VI’s clearly-labelled secondary row), not the transmitted amplitude this paper otherwise supports. No channel observationally contradicts $f_{\text{NL}}^{\text{local}} = -35/16$; the honest summary is three nulls (pulsar timing, PBH abundance, SMBH-seed abundance) and one channel (large-scale structure) that is reachable in principle but does not yet separate the two candidates.

B. Where the consistency is weak

The pulsar-timing channel is now a definite null rather than an open consistency: as Sec. IV D shows, this model’s own committed spectrum predicts $\gamma_{\text{pred}} \approx 5$, not the $\gamma = 3$ associated elsewhere in the literature with a small-scale-enhanced matter-bounce source [8] this model does not have – and it is identical at both candidate f_{NL} values regardless. The black hole channel now uses the literature’s own formation criterion and the lab’s own spectrum rather than an external, unreconstructible one, and returns a definite verdict (null) rather than an open amplitude scan; the amplitude ratio remains the channel’s only shape-robust, spectrum-independent output, and does not by itself separate $-35/16$ from $-35/8$ observationally – only compares how hard each is to produce – and $f_{\text{PBH}}(f_{\text{NL}})$ itself was shown to be strongly non-monotonic along the path to both candidates on the earlier lognormal grid (Sec. V B). The transmission result is scheme-qualified and validated only for $k\eta_B \lesssim 10^{-2}$; scheme-independence, previously an open question, is now answered negatively rather than left unresolved: the effective-fluid scheme S2, evaluated on its raw (pre-integration-by-parts) cubic action with exact mode functions, is finite – the earlier “S2 diverges” statement traced to a total-derivative artefact, not a physical pathology – but gives $f_{\text{NL}}^{\text{after}} \approx -1.25$ on the one background tested (Quintin-type), a factor ≈ 2.5 from S1’s -0.50 on the same background (Sec. III A). The transmitted amplitude is therefore reported as a two-scheme band rather than a single scheme-S1 number, and which scheme’s linear variable is physical at $H = 0$ remains an open theory question this paper does not resolve. At the transmitted amplitude the survey channel’s discriminating power is itself the weak link: SPHEREx separates the two candidates by $|\Delta f_{\text{NL}}^{\text{after}}| \approx 0.35\text{--}0.55$ against $\sigma_{f_{\text{NL}}} = 0.5\text{--}0.7$, i.e. $0.5\text{--}1.1\sigma$ (Sec. VI), so no currently forecast LSS survey can distinguish $-35/16$ from $-35/8$ once scheme-S1 transmission is applied; a not-yet-derived shape-overlap projection $r_{\text{sh}} < 1$ would only narrow this separation further, and since r has not been computed at this paper’s fiducial no quantitative statement about its effect is made here.

C. Next steps

In priority order: (i) settling which linear variable ($z = a$ or the effective-fluid $z^2 \propto \epsilon/c_s^2$) is the physically correct continuation through $H = 0$ – a theory question, not a numerical one, since both schemes are now finite and give genuinely different answers rather than differing by a regularization choice (Sec. III A); (ii) an evaluation-time (freeze-out) prescription for the bispectrum at $k\eta_B \gtrsim 0.1$, where this model’s absence of a post-bounce inflationary phase leaves ζ unfrozen and the bounce-window cubic integral dependent on when it is evaluated by a factor of several, so that the $k\eta_B \approx 0.6\text{--}0.8$ transfer feature and

the $k\eta_B \sim 1\text{--}3$ exact-mode bispectrum of Sec. III can be quoted as single numbers rather than ranges; (iii) re-derivation of the shape overlap r at the $-35/16$ fiducial; (iv) extending the S2 raw-ADM computation to the LQC and poly backgrounds, and to the $-35/8$ amplitude, neither of which is computed here (Sec. III A, Table VI); and (v) a direct-collapse (rather than horizon-re-entry) primordial-black-hole formation channel during the dust contraction phase itself, identified but not evaluated in Sec. V C. The propagation of the post-bounce amplitude to the CMB/LSS pivot and of the scalar spectrum through the induced-GW kernel to the nHz amplitude are carried out in Secs. VI and IV D; the bounce-scale enhancement mechanisms at $k\eta_B \sim 1$ that would be the remaining route to reopen Channel I or the black-hole channel have been checked directly (velocity dip, transfer excursion, and LQC cubic-operator enhancement, Sec. III) and found not to reopen either. Two further items follow directly from Secs. VII–VIII: (vi) converting the curvaton route of Sec. VIII into a concrete, checkable prediction requires an entropy-sector kinetic-amplification mechanism realized on the A2 bounce backgrounds used throughout this paper, and a demonstration that σ_* stays constant across them – neither is modelled here, and Cai, Brandenberger & Zhang’s [25] own ζ_{NL} integral through deflation and bounce remains un-re-derived by this lab; and (vii) the consistency relation $n_T = n_s - 1 = -0.035$ (Sec. VII) is a surviving, cheap discriminator of this class of background from single-field inflation independent of the (r, f_{NL}) no-go, worth a dedicated forecast. The general- P_{XXX} coefficient λ is no longer an open item: Sec. III A shows $r = 24c_s$ and $\Delta f_{\text{NL}}^{\text{bounce}}(c_s)$ are exactly λ -independent, so the no-go holds for the full $P(X)$ class (row19_lambda, reproducibility statement below).

Appendix A: The $\delta N_c - \zeta_{\text{Mald}}$ derivation

This appendix transcribes, without new science, the derivation archived in the monopole-adjudication note cited in the reproducibility statement below (§§1–4) that underlies Eq. (5) and the $[L]/[K]/[X]/[S]$ decomposition of Sec. IID.

1. The linear-order identity, Eq. (5)

On a uniform- ϕ slice the lapse is $N = 1 + \dot{\zeta}/H$, so the Friedmann equation at fixed ϕ gives $\delta H/H = -(\epsilon/3)\dot{\zeta}_L/H$: the local expansion rate on comoving slices is perturbed at $O(k^0)$, carried by the trace of the shift gradient, $\delta K = -\partial^2\psi/a^2 \rightarrow -\epsilon\dot{\zeta}_L$. The separate-universe δN is defined on the zero-shift threading, $\delta N_c = \int (K/3) d\tau$, so writing $\delta N_c = \zeta + b$ with $\dot{b} = \delta K/3$ gives, on the growing mode, $b = -(\epsilon/3)\dot{\zeta}_L$, i.e. Eq. (5), $\delta N_c = (1 - \epsilon/3)\zeta_{\text{Mald}}$ at linear order (sign convention: ζ_{Mald} and δN_c share the sign of the comoving-gauge curvature perturbation used throughout this paper).

2. The $[L]/[K]/[X]/[S]$ decomposition

Separating the long mode's coupling to the short-mode pair by how it enters:

class	long mode enters as	squeezed value at $\epsilon = 3/2$
$[L]$	$\zeta_L, \dot{\zeta}_L$ (lapse/curvature; local response)	$-25/8$, pure monopole
$[K]$	$\partial_i \partial_j \tilde{\chi}_L$ (δK and shear)	0 (cancels between varied legs of T_4)
$[X]$	$\partial_i \tilde{\chi}_L$ (the <i>shift</i> , $\propto 1/k_L$)	$\frac{15}{16} + \frac{15}{16}\mu^2$ (monopole $5/4$)
$[S]$	short-mode sourcing of the long mode	0 ($O(k_L^2/a^2 H^2)$ -suppressed)
total		$-\frac{35}{16} + \frac{15}{16}\mu^2$ (= in-in exactly)
comoving δN (lanes A/B/C)	different variable, no $[X]$	-5 , isotropic

The $1/k_L$ poles in $[X]$'s two contributions – the T_3 bulk coupling and the boundary term f_b below – cancel only in their sum, leaving a finite $O(k_L^0)$ remainder; no separate-universe construction (isotropic or Bianchi-I) contains this term, because it originates in the comoving-gauge *shift* $N_i = \partial_i \psi \supset a^2 \epsilon \partial^{-2} \zeta$, which is $O(1/k_L)$ in this non-attractor phase and so violates the gradient-expansion assumption $N_i = O(\nabla)$ every separate-universe construction relies on. The final-time re-threading boundary term,

$$f_b = \frac{\epsilon}{2\mathcal{H}} \left[\partial \zeta \partial \tilde{\chi} - \partial^{-2} \partial_i \partial_j (\partial_i \zeta \partial_j \tilde{\chi}) \right], \quad (\text{A1})$$

subtracts $(1 - \epsilon/3) \times \frac{5\epsilon}{4} \mu^2$ from the T_3 bulk's μ^2 coupling and adds the $+15/16$ monopole, reproducing the μ^2 coefficient exactly and supplying the $[X]$ monopole. In-in monopole minus δN_c : $-\frac{15}{8} - (-5) = \frac{25}{8} = [X]_{\text{mono}} (5/4) + ([L] - \delta N_c) (15/8)$; in general ϵ , $5\epsilon(9 - \epsilon)/18 = 5\epsilon(9 - 2\epsilon)/36 + 5\epsilon/4$. The zero-shift, fluid-worldline threading identity underlying δN_c is $\delta N_c(x_f) = \zeta(t_f, x_f) - \frac{1}{3} \int \partial_i N^i dt$ along the worldline; expanding it to second order at constant ϵ gives $f_{\delta N_c} = f^{\text{in-in}}/\lambda + f_{\text{map}}$ with $\lambda = 1 - \epsilon/3$. The worldline can be labelled by its position on the *initial* slice (the label the separate universe uses, since it lays out its patches there) or on the *final* slice; the two maps are

$$f_{\text{map}}^{\text{init}} = \frac{5\epsilon}{4(3 - \epsilon)} [(\epsilon - 2) - \epsilon\mu^2], \quad f_{\text{map}}^{\text{fin}} = -\frac{5\epsilon}{4} (1 - \mu^2), \quad (\text{A2})$$

each with monopole $-5\epsilon/6$. The *initial-label* composition, $f_{\delta N}^{\text{init}} = f^{\text{in-in}}/\lambda + f_{\text{map}}^{\text{init}}$, is an isotropic monopole of exactly -5 for every constant ϵ , not a numerical coincidence; this is the label the separate universe computes. The *final-label* composition, $f_{\delta N}^{\text{fin}} = f^{\text{in-in}}/\lambda + f_{\text{map}}^{\text{fin}} = -\frac{25}{4} + \frac{15}{4}\mu^2$ at $\epsilon = 3/2$, differs by a translation term with no monopole; the monopole gap to the in-in value is label-independent. (An earlier draft of this appendix

printed $f_{\text{map}}^{\text{fin}}$ composed with the initial-label total -5 , evaluating to $-\frac{25}{4} + \frac{15}{4}\mu^2$ under the initial-label name; both numbers above are correct, the label was mixed.) The relation between δN_c and ζ_{Mald} is an exact, invertible change of variable ($f_{\delta N_c} = f^{\text{in-in}}/\lambda + f_{\text{map}}$, $\lambda \neq 0$); the $O(1)$ effect above is an error only in the standard δN formula's identification $\delta N_c = \zeta$, and it arises iff the neglected super-Hubble integral $I = \zeta_{L,f}^{-1} \int_{t_i}^{t_f} (\epsilon/c_s^2) \dot{\zeta}_L dt$ is $O(1)$ – not from any physical translation mechanism.

3. General- ϵ formulas

The comoving separate-universe results, for general ϵ , are

$$f_{\text{NL}}^p = \frac{5(\epsilon - 7)}{8}, \quad f_{\text{NL}}^c = -5 \quad (\text{all } \epsilon), \quad (\text{A3})$$

and the in-in squeezed bispectrum, general ϵ , is

$$f^{\text{sq}}(\mu, \epsilon) = \frac{5}{12} (\epsilon^2 \mu^2 - \epsilon^2 + 6\epsilon - 12), \quad (\text{A4})$$

with monopole $-5(\epsilon - 3)(\epsilon - 6)/18$, isosceles ($\mu = 0$) value $-5(\epsilon^2 - 6\epsilon + 12)/12$, and μ^2 coefficient $5\epsilon^2/12$; at $\epsilon = 3/2$ these reduce to the monopole $-15/8$, the isosceles value $-35/16$ of Eq. (4)'s $\mu = 0$ limit, and the $+15/16 \mu^2$ coefficient (Legendre $\ell = 2$ coefficient $5/8$) quoted throughout this paper. These general- ϵ expressions hold on the non-attractor growing branch ($\dot{\zeta} = -3\zeta/\eta$) that this paper's entire construction uses, and do *not* reduce to the slow-roll attractor (Maldacena) result $(5/12)(1 - n_s) \rightarrow 0$ as $\epsilon \rightarrow 0$; they are not claimed to.

4. Bianchi-I: why an anisotropic separate universe does not close the gap

An anisotropic (Bianchi-I) separate-universe treatment, carrying the long mode's projected shear onto the short-mode power spectrum exactly, gives an identically zero monopole contribution: the shear is $\mathcal{O}(k_L^0)$ and traceless, and no traceless linear-in- ζ_L response can supply a monopole. It therefore cannot by itself close the $-25/8$ gap between $\delta N^c = -5$ and the in-in monopole $-15/8$; the gap is closed only by $[X]$, the shift term no separate-universe construction contains (Sec. A2 above). A direct second-order continuation of the uniform-density (-slice) δN variable itself – as opposed to a Bianchi-I correction to the isotropic construction – gives $f_{\text{NL}}^p = 5(2\epsilon - 15)/24 = -5/2$ at $\epsilon = 3/2$ (isotropic, initial label); this does not reproduce the separate-universe value $-55/16$ quoted in Sec. IID (gap $15/16$). The intrinsic term, $f_{\text{NL}}^{\text{intr}} = \frac{5\sqrt{3}\epsilon(3-\epsilon)}{18} \frac{r_i}{W-1}$, vanishes in the growing-mode limit that defines $-55/16$ for any t_f -independent r_i , so it does not close this gap. An independent exact separate-universe solution reproduces both $-55/16$ (uniform-density slice) and -5 (comoving slice) for all ϵ , so the

uniform-density δN value is itself well-defined, not an evolution-step artifact; the gap $5(6 - \epsilon)/24$ is closed, per an independent adjudication (`research/theory_audit/a2.lapse_monopole_adjudication_2026.09.07.md`), by correcting the composition weight on the second-order curvature perturbation from $\lambda' = 2\lambda$ to 3λ : the threading map's long $\times$ short lapse monopole coefficient $A_2 = \epsilon(3 - \epsilon)^2/3$ (this map) is the correct constraint-solve value, not the separate universe's $2(3 - \epsilon)^2$. Both the comoving-slice δN value (-5) and the uniform-density δN value ($-55/16$) are now fully reconciled by the map derived here; the flagship in-in monopole $-35/16$ is unaffected.

REPRODUCIBILITY STATEMENT

Every result reported here traces to a committed script, chain, or reproducibility manifest; nothing is quoted from memory. The exact-amplitude derivation (Sec. II) and the bounce-transmission bound (Sec. III) are committed, pinned to commit 68309c8, at

https://github.com/Hubify-Projects/bigbounce/tree/68309c8/research/theory_audit

https://github.com/Hubify-Projects/bigbounce/tree/68309c8/research/cubic_bounce_transmission

respectively, including `fnl_monopole_adjudication_2026.09.03.md` in the former (Appendix A). The ledger-row-9 (D-A3-9) mechanisms of Sec. III and the S2 resolution of Sec. IIIA are committed, pinned to commit 68309c8 (the same pointer used for Secs. II–IID), under `research/cubic_bounce_transmission/`: `lane9a_velocity_dip/` (Quintin Eq. 79 evaluation and the $k\eta_B \in [0.1, 10]$ transfer scan), `lane9b_s2_regulation/` (the total-derivative diagnosis of the S2 divergence), `lane9b2_s2_rawadm/` (the raw-ADM S2 computation, `results.json`), `lane9c_abs_operator/` (the Agullo–Bolliet–Sreenath operator map and scale-window conversion), and `lane9c2_lqc_modes/` (the exact-mode LQC-dust in-in evaluation, `results.json`), each with its own manifest under `reproducibility/manifests/experiments/`. The tensor $H = 0$ continuation of Sec. VII is at `research/cubic_bounce_transmission/row18a_s2_tensor/` (`results.json`); the c_s -dependent bounce cubic term of Sec. IIIA is at `research/cubic_bounce_transmission/row18b_cs_bounce_cubic/` (`row18b_cs_bounce_cubic.py`, `results.json`). The general- λ extension of Sec. IIIA (the λ -independence of r and $\Delta f_{\text{NL}}^{\text{bounce}}$, and the λ/Σ scan) is at `research/track_a3_multichannel/row19_lambda/` (`row19_lambda.py`, `results.json`, `row19_lambda.log`), manifest `reproducibility/manifests/experiments/a3-row19-lambda.json`. The bounce-to-local shape overlap ($r_{\text{sh}} = 1$, an assumed value pending an explicit shape-overlap projection; Sec. VI) is taken from this lab's companion f_{NL} -forecast program (`outputs/survey_reach_fnl.json`); every tabulated survey significance in Table VI is accordingly an upper

bound on the detectable significance, not a forecast of it. The three channel computations of Secs. IV–VI are committed at `research/track_a3_multichannel/`: `pta_gamma_reproduce.py`, `pbh_abundance_fnl.py` (first-pass Press–Schechter, retained for comparison only), `pbh_compaction_fnl.py` (compaction-function criterion, this paper's principal PBH result), `inlab_delta2_zeta_2026-09-03.py` (Sec. VC, the lab's own predicted spectrum and its FIRAS check), `survey_reach_fnl.py`, `pta_injection_30bin_2026.09.02.py`, and `pta_injection_30bin_realkde_2026.09.02.py` (the Sec. IVC injection-recovery validations), and `PBH_COMPACTON_NOTE_2026-09-02.md` (the Sec. VB compaction-note validation), and `sigw_nhz_from_lab_spectrum_2026.09.04.py` (Sec. IVD, the induced-GW SIGW computation), with results in `outputs/*.json` and manifests at `reproducibility/manifests/experiments/a3-*.json` (PTA/PBH/reach, including `a3-pbh-compaction-fnl.json`), `reproducibility/manifests/experiments/p2-fnl-*.json` (exact-amplitude derivation and validation), and `reproducibility/manifests/experiments/p2-a2-*.json` (bounce-transmission computation). The pulsar-timing chain is `pipelines/p3_pta_mcmc/free_spectrum_real_2026-05-01/chain_real_freespec.npy` (the directory date is this lab's run date of the reduction, not the underlying NANOGrav release date) (SHA-256, in two halves to allow line-breaking: `50abc38a04e1bf886adc833b5eb653be4e986877fe2476f87a78927f4f3610fc`), derived from the NANOGrav 15-yr KDE free spectra, Zenodo 10.5281/zenodo.8060824. This work's own code and artifacts are pinned by the GitHub commit hash above rather than a frozen-release DOI of their own; minting one is a maintainer action planned for the packaging stage. The exact-amplitude theory lineage this paper builds on is archived at Zenodo 10.5281/zenodo.21461881. The DESI DR1 QSO reproduction of Sec. VI is committed at `research/desi_png_reproduction/`, manifest `LEDGER4_RESULT.v3_2026-09-04.md`. The tensor amplitude of Sec. VII (ledger row 10), including the $\gamma_{\text{pred}}^{(1)} = 5.035$ nHz propagation of Sec. IVD (`outputs/r5_15_tensor_omega_nhz.json`), is committed at `research/track_a3_multichannel/row10_r_ns/` (`row10_r_ns.py`, `results.json`); the joint (r, f_{NL}) no-go of Sec. VIII (ledger row 14) is at `research/track_a3_multichannel/row14_cs_window/` (`row14_cs_window.py`, `results.json`); the curvaton-route adjudication is at `research/theory_audit/curvaton_matter_bounce_adjudication_2026.09.04.py`; the PBH sign-correction and extended γ_{cr} scan of Sec. VB (ledger row 11), including `outputs/r11_pbh_residuals.json`,

is at `research/track_a3_multichannel/row11_pbh_residuals/`; and the Appendix A threading-map identity is at `research/theory_audit/threading_map_second_order_2026.09.04.py`, each with a manifest under `reproducibility/manifests/experiments/`. Every computation reported in this paper ran on a local CPU (Apple M5, 24 GB, macOS 26.5) at zero compute cost; the PTA/PBH/reach/injection channels ran in under 6s total. Compute venue, wall-clock time, and external data sources are recorded per-experiment in the manifests cited above.

AI USAGE DISCLOSURE

Symbolic derivations reported in Secs. II–III were performed by deterministic sympy scripts written and ex-

ecuted under the author’s direction; every intermediate result is printed by the committed scripts and independently checked against the cited literature limits before use. The channel computations of Secs. IV–VI were performed by committed Python scripts against real data and cited published values, not by an LLM. Claude (Anthropic) assisted with literature cross-checks, prose drafting, and L^AT_EX formatting under the author’s direction; all physics content, equations, and numerical results were verified by the author against the committed script output.

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- [1] Y.-F. Cai, W. Xue, R. Brandenberger, and X. Zhang, “Non-Gaussianity in a Matter Bounce,” JCAP **0905**, 011 (2009), arXiv:0903.0631.
 - [2] Y.-F. Cai, D. A. Easson, and R. Brandenberger, “Towards a Nonsingular Bouncing Cosmology,” JCAP **1208**, 020 (2012), arXiv:1206.2382.
 - [3] J. Quintin, Z. Sherkatghanad, Y.-F. Cai, and R. H. Brandenberger, “Evolution of cosmological perturbations and the production of non-Gaussianities through a nonsingular bounce: Indications for a no-go theorem in single field matter bounce cosmologies,” Phys. Rev. D **92**, 063532 (2015), arXiv:1508.04141.
 - [4] Y.-B. Li, J. Quintin, D.-G. Wang, and Y.-F. Cai, “Matter bounce cosmology with a generalized single field: non-Gaussianity and an extended no-go theorem,” JCAP **1703**, 031 (2017), arXiv:1612.02036.
 - [5] J. Maldacena, “Non-Gaussian features of primordial fluctuations in single field inflationary models,” JHEP **05**, 013 (2003), arXiv:astro-ph/0210603.
 - [6] M. H. Namjoo, H. Firouzjahi, and M. Sasaki, “Violation of non-Gaussianity consistency relation in a single field inflationary model,” Europhys. Lett. **101**, 39001 (2013), arXiv:1210.3692.
 - [7] I. Agullo, B. Bolliet, and V. Sreenath, “Non-Gaussianity in loop quantum cosmology,” Phys. Rev. D **97**, 066021 (2018), arXiv:1712.08148.
 - [8] T. Papanikolaou, “Gravitational wave signatures of non-singular matter bouncing cosmology in NANOGrav and beyond,” arXiv:2504.11641.
 - [9] Y. Cai, S. Pi, and M. Sasaki, “Universal infrared scaling of gravitational wave background spectra,” Phys. Rev. D **102**, 083528 (2020), arXiv:1909.13728.
 - [10] K. Kohri and T. Terada, “Semianalytic calculation of gravitational wave spectrum nonlinearly induced from primordial curvature perturbations,” Phys. Rev. D **97**, 123532 (2018), arXiv:1804.08577.
 - [11] G. Agazie *et al.* (NANOGrav Collaboration), “The NANOGrav 15 yr Data Set: Evidence for a Gravitational-wave Background,” Astrophys. J. Lett. **951**, L8 (2023), arXiv:2306.16213.
 - [12] NANOGrav Collaboration, “NANOGrav 15 yr KDE Free Spectra v1.0.0,” Zenodo, [10.5281/zenodo.8060824](https://doi.org/10.5281/zenodo.8060824).
 - [13] S. Young and C. T. Byrnes, “Primordial black holes in non-Gaussian regimes,” JCAP **1308**, 052 (2013), arXiv:1307.4995.
 - [14] G. Franciolini, A. Kehagias, S. Matarrese, and A. Riotto, “Primordial Black Holes from Inflation and non-Gaussianity,” JCAP **1803**, 016 (2018), arXiv:1801.09415.
 - [15] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, “Primordial black holes – perspectives in gravitational wave astronomy,” Class. Quant. Grav. **35**, 063001 (2018), arXiv:1801.05235.
 - [16] S. Choudhury, K. Dey, S. Ganguly, A. Karde, S. K. Singh, and P. Tiwari, “Negative non-Gaussianity as a salvager for PBHs with PTAs in bounce,” Eur. Phys. J. C **85**, 472 (2025), arXiv:2409.18983.
 - [17] S. Young, C. T. Byrnes, and M. Sasaki, “Calculating the mass fraction of primordial black holes,” JCAP **07**, 045 (2019), arXiv:1904.00984.
 - [18] I. Musco, “Threshold for primordial black holes: Dependence on the shape of the cosmological perturbations,” Phys. Rev. D **100**, 123524 (2019), arXiv:1809.02127.
 - [19] E. Chaussidon *et al.* (DESI Collaboration), “Constraining primordial non-Gaussianity with DESI 2024 LRG and QSO samples,” arXiv:2411.17623.
 - [20] C. Heinrich, O. Doré, and E. Krause, “Measuring f_{NL} with the SPHEREx Multi-tracer Redshift Space Bispectrum,” arXiv:2311.13082.
 - [21] O. Doré *et al.*, “Cosmology with the SPHEREx All-Sky Spectral Survey,” arXiv:1412.4872.
 - [22] S. Ferraro *et al.*, “Inflation and Dark Energy from spectroscopy at $z > 2$,” Astro2020 science white paper, arXiv:1903.09208.
 - [23] N. Sailer, E. Castorina, S. Ferraro, and M. White, “Cosmology at high redshift – a probe of fundamental physics,” JCAP **12**, 049 (2021), arXiv:2106.09713.
 - [24] P. A. R. Ade *et al.* (BICEP/Keck Collaboration), “Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season,” Phys. Rev. Lett.

- 127**, 151301 (2021), arXiv:2110.00483.
- [25] Y.-F. Cai, R. Brandenberger, and X. Zhang, “The Matter Bounce Curvaton Scenario,” JCAP **03**, 003 (2011), arXiv:1101.0822.
- [26] D. H. Lyth, C. Ungarelli, and D. Wands, “The Primordial Density Perturbation in the Curvaton Scenario,” Phys. Rev. D **67**, 023503 (2003), arXiv:astro-ph/0208055.
- [27] M. Sasaki, J. Valiviita, and D. Wands, “Non-Gaussianity of the Primordial Perturbation in the Curvaton Model,” Phys. Rev. D **74**, 103003 (2006), arXiv:astro-ph/0607627.
- [28] Planck Collaboration (Y. Akrami *et al.*), “Planck 2018 Results. IX. Constraints on Primordial Non-Gaussianity,” Astron. Astrophys. **641**, A9 (2020), arXiv:1905.05697.