

Survey-Scale Galaxy Chirality with Equivariant TTA: A Null Real-Space Chirality Dipole, a Quantifiable Monopole-Mask Leakage Channel, and Diagnostic Evidence for a Depth/Morphology-Correlated Canonical-Mask Residual on 8.47 Million DESI Legacy Galaxies (3.2 Million Spirals)

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We present, to our knowledge, the largest chirality-labeled galaxy catalog to date: 8,474,531 DESI Legacy DR8 galaxies classified by a flip-equivariant Vision Transformer pipeline into clockwise (CW), counter-clockwise (CCW), and non-spiral classes, with $N_{\text{spiral}} = 3,201,160$ spirals, publicly released with model weights and reproducibility scripts. The primary scientific result is a real-space chirality dipole consistent with null: the equivariant-catalog high-confidence dipole fit (confidence > 0.6 , a pre-specified selection threshold — not tuned post-hoc — under which the null is robust across the high-confidence regime ($p_{\text{eq}} \in \{0.6, 0.7, 0.8\}$) of a full confidence-cut sweep, while the low-confidence tail ($p_{\text{eq}} \leq 0.5$) shows a systematics-attributed excess ($z \approx 4.0\text{--}4.3$), Secs. III B, IV C; $N \approx 9.5 \times 10^5$ spirals) gives $+0.41\sigma$ (moment- z against the isotropic (pixel-)permutation null; empirical-rank $p = 0.31$, 10^4 isotropic-null realizations; robust under a per-galaxy label-shuffle null, $z = 0.58$ (same-generator primary; an independent re-implementation gives $z = 0.70$), and the unthresholded-sample sensitivity is attributed to a low-confidence-tail systematic in Sec. IV), and a block-bootstrap WLS template fit disfavors a clean cosmological dipole at the 1.7% reference amplitude (the lower end of Shamir’s reported 1.7%–4.0% asymmetry range [1, 3]; this 1.7% f_{CW} -asymmetry maps to $A_{\text{ref}} = 0.034$ in A_p units, since $A_p = 2(f_{\text{CW}} - 0.5)$) at $z \approx -18$ under the adopted NSIDE=8 block-bootstrap error model (Appendix D). *Note: the $+0.41\sigma$ (isotropic (pixel-)permutation null) and $z = 0.58$ (per-galaxy label-shuffle null, same-generator primary) values arise from distinct null procedures and are diagnostic-only, not directly comparable as detection significances.* This $\ell = 1$ observable is parity-even (isotropy-breaking axial-vector channel), not a direct parity-violation test. The MASTER pseudo- C_ℓ channel on the patchy footprint is a systematics diagnostic, not an independent cosmological null: a monopole-only generative null reproduces 99.32% of the raw pre-MASTER $\ell = 1$ power (monopole-mask leakage), and MASTER deconvolution substantially reduces, but does not remove, this leakage — the post-MASTER harmonic diagnostics carry systematics-attributed residuals ($+3.64\sigma$ moment- z , $\approx 1.9\sigma$ Gaussian-equivalent, canonical mask; $+7.28\sigma$, apodized footprint) that are diagnostic quantities attributed to residual survey systematics, *not* claimed cosmological detections, characterized by an eight-anchor systematic battery (Appendix D). (The $+3.64\sigma$ value is from a 500-MC direct run on the canonical unapodized mask; the 10^4 -permutation canonical unapodized row in Table IV gives $+7.93\sigma$; both are systematics-attributed diagnostics from different null-run sizes and mask/weight conventions (Sec. III A), not two independent detection claims. *Note: the σ values quoted in this paragraph arise from distinct null procedures — see Sec. III A and Table IV — and are not directly comparable as detection significances; they are diagnostic indicators only.*) Falsification criterion: a future $\geq 5\sigma$ real-space dipole detection (moment- z vs. the isotropic (pixel-)permutation null) at amplitude $A \gtrsim A_{95}$, where injection-recovery brackets A_{95} between 1.0% and 1.5% ($A_{50} \approx 0.75\%$), would be in tension with the present null. These thresholds are estimator-specific to the real-space dipole; the harmonic-channel completeness ($P(\geq 3\sigma) \geq 0.999$ at $A_p = 0.75\%$, where the 3σ refers to the MASTER $\ell = 1$ label-shuffle null, a different convention) is a separate diagnostic property of that channel and is not interchangeable with the real-space falsification boundary.

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I. INTRODUCTION

The handedness (chirality) of spiral galaxies—whether their arms trail clockwise (CW) or counter-clockwise (CCW) as projected on the sky—is a simple observable that, under the trailing-arm assumption and in the absence of confounding selection effects, traces the angular-momentum direction of each disk galaxy. Throughout this paper, “CW/CCW” refers to the *projected apparent arm-winding chirality*, not a deprojected 3D spin vector. In a statistically isotropic and parity-symmetric universe, the CW and CCW fractions should be exactly equal when averaged over large angular scales. A significant directional departure would constrain isotropy-breaking axial-vector sectors in the galaxy-formation chain. The present paper is a standalone observational result: our null dipole at sub-percent sensitivity does not depend on any unpublished companion work.

Claims of such a signal have appeared intermittently in the literature. Shamir (2012) [4] reported a 2–4 σ dipole with per-bin asymmetry amplitudes of ~ 5 –20% (as reported in that work) using $\sim 1.27 \times 10^5$ SDSS

galaxies. Shamir (2020) [1] reported asymmetries at the reported ~ 2 –4% level on SDSS and Pan-STARRS samples; Shamir (2022a) [2] reported related spin-direction alignment analyses; and Shamir (2022b) [3] reported results on a DESI Legacy sample (“nearly 1.3×10^6 spiral galaxies” per the published abstract). Iye *et al.* (2021) [5] re-examined Shamir’s SDSS spiral catalog using 3D random-walk simulations and found no significant dipole after correcting for reading-direction bias and photometric-object duplication in earlier Shamir catalogs. Tadaki *et al.* [6] likewise found null results on a catalog of $\sim 80,000$ face-on spirals. Jia *et al.* [7] introduced CE-ResNet, a chirality-equivariant CNN guaranteeing by construction that flipping an input exactly swaps CW and CCW outputs, with a reported number-count ratio $\text{CW}/\text{CCW} = 0.998$ on ~ 1.95 million galaxies.

In this paper we present a new chirality catalog whose novelty relative to CE-ResNet lies in scale and bias-hardening rather than classifier accuracy: (i) survey-scale coverage of 8.47 million galaxies (3,201,160 equivariant-classified spirals, $1.6 \times$ CE-ResNet’s scale); (ii) a dedicated NOT_SPIRAL class preventing contamination from ellipticals and irregulars; and (iii) a multi-axis bias-hardening audit suite. (A substantial fraction of our training labels derive from CE-ResNet predictions, so the two catalogs are not fully independent; Sec. II and Appendix B quantify the independent GZ1 agreement. A corollary limitation: the label-shuffle and per-pixel permutation nulls used throughout randomize this model’s own outputs, so they do not by themselves test independence from any large-scale survey-correlated structure potentially inherited through the CE-ResNet pseudo-labels; that axis is constrained instead by the template-regression and cross-spectrum diagnostics of Appendix D.) We use this catalog to perform a chirality dipole measurement with an empirical 50%-recovery-3 σ injection-recovery threshold at $|A_{\text{dipole}}| \geq 0.75\%$. The measured dipole is consistent with null: the equivariant CW fraction is 0.4974 ± 0.000279 and the real-space dipole significance is $+0.41\sigma$ ($p = 0.31$; primary), with a block-bootstrap WLS template fit disfavoring a clean 1.7% dipole at $z \approx -18$; the MASTER pseudo- C_ℓ channel on the patchy footprint is systematics-dominated and serves only as a diagnostic — its post-MASTER residuals ($+3.64\sigma$ canonical, $+7.28\sigma$ apodized) are attributed to residual survey systematics and are *not* detections (Secs. IV C, IV D). This is inconsistent in amplitude with Shamir’s claimed $\sim 3\%$ signal by a factor of ~ 4 –9 under the present pipeline (comparing the canonical joint nuisance-marginalized WLS best-fit amplitude 0.455% in A_p units, Appendix D, to Shamir’s 1.7%–4.0% reported range), though a matched-footprint Galyzer reanalysis is required for a likelihood-level exclusion.

II. DATA

A. Galaxy Images

Our parent sample is the Smith42/galaxies dataset on HuggingFace (<https://huggingface.co/datasets/Smith42/galaxies>), containing 8,474,688 galaxy images from the DESI Legacy Imaging Surveys DR8 [8]. Each image is a 224×224 pixel cutout in *grz* bands at $0.262''/\text{pixel}$. The dataset includes unique `dr8_id` identifiers; sky coordinates are obtained by cross-matching against the Galaxy Zoo DESI predictions catalog [9]. The parent-sample selection function inherits from Galaxy Zoo DESI: photometric types REX/DEV/EXP/SER, $r < 19.0$, half-light radius $\geq 3''$. DR8 comprises three distinct imaging campaigns: BASS+MzLS ($\delta > +32^\circ$), DECaLS ($\delta < +32^\circ$), and a DES overlap region.

B. Training Labels

We assemble training labels from three sources: (1) Galaxy Zoo 1 [10]: 6,637 galaxies with CW/CCW labels at $> 70\%$ vote confidence; (2) CE-ResNet [7]: 17,153 galaxies with high-confidence spiral classifications; (3) Synthetic hard negatives: 2,000 artificial images as NOT_SPIRAL training examples. The combined training set contains $6,637 + 17,153 + 2,000 = 25,790$ source images; after flip augmentation of the training split the combined pool is 26,616 images (pre-augmentation 79.4/20.6 source-image split: $n_{\text{train}} = 21,293$ post-augmentation, $n_{\text{val}} = 5,323$ never augmented; the 826-image difference between the source manifest (25,790) and the combined pool (26,616) arises entirely from horizontal-flip augmentation applied to the *training split only* — the validation split ($n_{\text{val}} = 5,323$) is never augmented; Appendix B, artifact [pipelines/p2_chirality/outputs/canonical_provenance/c17_item13_training_semantics.json](#)). Note: $17,153/25,790 = 66.5\%$ of training labels derive from CE-ResNet predictions; validation metrics against the full training set therefore partially reflect agreement with CE-ResNet rather than independent ground truth. The independent GZ1 cross-match on 234,282 disjoint matches yields spiral-chirality accuracy 69.91% (Cohen’s $\kappa = 0.40$). We treat 69.91% as an explicitly conservative accuracy floor and propagate it to all downstream isotropy bounds via the sub-percent systematic floor in Sec. IV C; because a moderate-accuracy classifier dilutes any real chirality dipole toward null (the dilution $g = 2a - 1$ is folded into the empirical injection-recovery floors of Sec. VIB, not assumed away), this floor renders the present null detection conservative rather than optimistic.

III. METHODS

A. Notation and Significance Conventions

a. Confidence notation. p_{eq} (or $p_{\text{CW}}^{\text{eq}}$, $p_{\text{CCW}}^{\text{eq}}$, $p_{\text{NS}}^{\text{eq}}$) denotes the max-class equivariant probability output by the 2-fold flip-TTA pipeline (Sec. III D and Appendix B). These are monotone ranking scores, not frequentist probabilities; “high-confidence” (HC) cuts on p_{eq} are sample-selection thresholds.

b. Significance conventions. All significance values z are moment-ratios $(x - \langle x \rangle_{\text{null}})/\sigma_{\text{null}}$ against the null distribution specified per result. Empirical rank p -values are one-sided unless labeled two-sided. Three distinct significance conventions appear in this paper:

- *Moment- z / rank- p :* the real-space dipole pair ($z_{\text{mom}} = +0.41$, rank- $p = 0.31$) are independent summary statistics of the A_{dip} distribution against the isotropic (pixel-)permutation null; they do not follow the Gaussian $z \rightarrow p$ mapping.
- *MASTER $\ell = 1$ moment- z :* $z = (C_1^{\text{data}} - \langle C_1 \rangle_{\text{null}})/\sigma_{\text{null}}$ vs. the label-shuffle null mean $\langle C_1 \rangle_{\text{null}}$ and width σ_{null} ; $z = +3.64$ (500-MC canonical direct run), $z = +7.93$ (10^4 -permutation canonical unapodized), $z = +7.28$ (10^4 -permutation apodized). These three values are from different null-run sizes and mask/weight conventions and are not mutually comparable.
- *Block-bootstrap z :* $(A_{\text{dipole}}^{\text{best}} - A_{\text{ref}})/\sigma_{\text{boot}}$ with spatial-coherence-corrected σ_{boot} ; the $z \approx -18.1$ primary exclusion uses this convention only.

B. Declared Analysis Hierarchy

We declare the estimator hierarchy used throughout. This hierarchy — with the real-space dipole as the row-(i) primary cosmological estimator — is documented in Appendix A; the primacy of the real-space estimator is established independently of the harmonic-channel diagnostics. The primary dipole measurement uses the high-confidence subsample (Catalog C with winning-class confidence $p_{\text{eq}} > 0.6$; $N_{\text{HC}} = 949,584$ spirals), which suppresses the depth-correlated low-confidence-tail systematic identified in Sec. IV C; all estimator-hierarchy and sensitivity statements below are made on this HC selection unless explicitly noted. See also Appendix D for the full eight-anchor discriminator table.

- **Primary cosmological estimators:** (i) real-space CW-fraction dipole fit on Catalog C at NSIDE=64 (moment-ratio $z_{\text{mom}} = +0.41$ against the isotropic (pixel-)permutation null, empirical-rank $p = 0.31$; these are independent numbers, not Gaussian (z, p) partners — see Sec. IV C); and (ii) block-bootstrap WLS template-fit exclusion of a clean 1.7% dipole (the lower end of Shamir’s

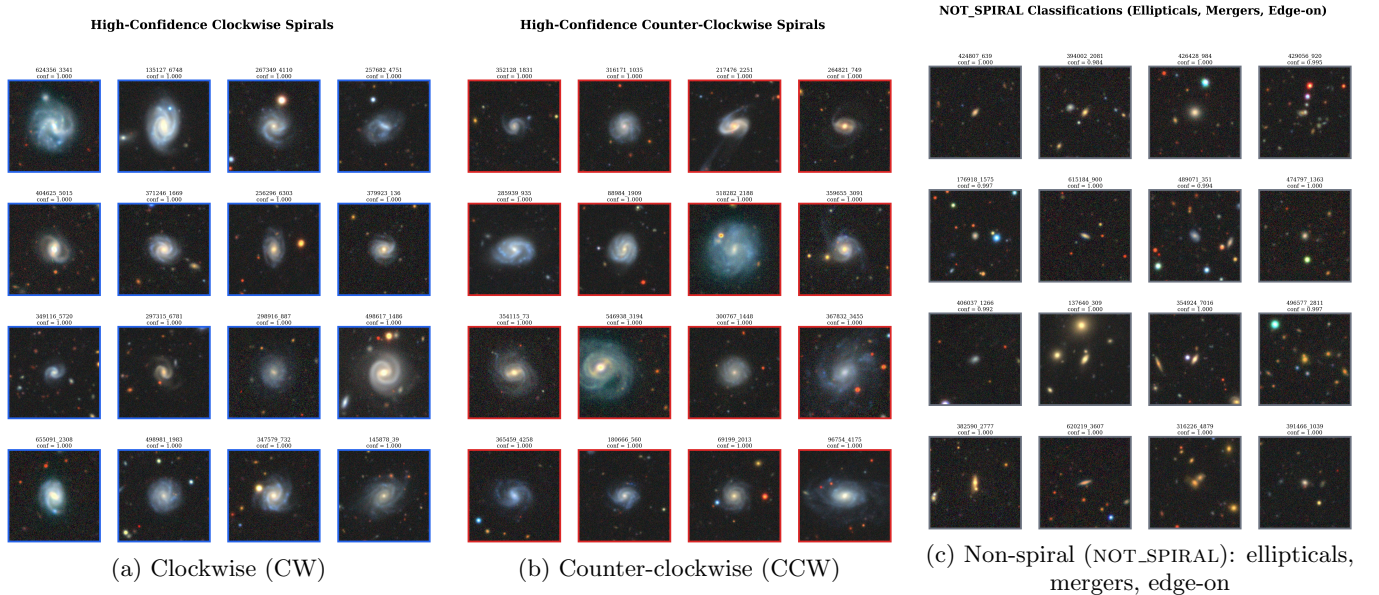


FIG. 1. **Representative high-confidence galaxies from the classified catalog** ($p_{\text{eq}} > 0.9$). Left: clockwise (CW) spirals; center: counter-clockwise (CCW) spirals; right: non-spiral (NOT_SPIRAL) objects — ellipticals, mergers, and edge-on galaxies that would contaminate a binary spiral classifier. All cutouts are 224×224 pixels in grz bands from DESI Legacy DR8. The gallery illustrates the visual diversity within each class and motivates the three-class design: without an explicit NOT_SPIRAL class, $\sim 62\%$ of the parent sample would leak into the spiral classification. The $ViT - Small$ classifier resolves CW vs. CCW via the test-time equivariant averaging procedure of §III D.

reported 1.7%–4.0% asymmetry range [1, 3]) on the canonical-mask A_p field ($z \approx -18$ under the adopted NSIDE= 8 block-bootstrap error model; Appendix D).

- **Secondary diagnostic estimators:** (iii) canonical- N direct-MC NaMaster at $\ell = 1$ on the patchy canonical mask ($f_{\text{sky}} = 0.49005$, $+3.64\sigma$); (iv) apodized-footprint MASTER at $\ell = 1$ ($N_{\text{all}} \geq 1$ mask, $f_{\text{sky}} = 0.494$, C^2 2°; $+7.28\sigma$ vs. global label-shuffle and $+7.13\sigma$ vs. depth-stratified null, $W_p = N_{\text{all}}$); and (v) hemisphere maximum-asymmetry (3.05σ local maximum against the label-shuffle null, $p_{\text{LEE}} \leq 10^{-4}$ direct-MC max-statistic, systematics-attributed (the Gaussian-Bonferroni $< 1\sigma$ heuristic is a non-principled cross-check); Appendix C). These characterize how a non-zero global monopole and coherent low- ℓ systematics couple through the patchy weighted footprint.
- **Generative monopole-only null:** (vi) $N = 500$ binomial-monopole realizations demonstrating that the raw pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ is dominated by monopole-mask leakage; the post-MASTER $+3.64\sigma$ canonical-mask residual is non-primary and requires additional coherent systematics beyond the monopole-only channel (Sec. IV D).
- **Sensitivity floor:** (vii) empirical injection-recovery on the HC-broad spiral subsample ($p_{\text{eq}} > 0.6$, $N = 949,584$; Sec. VIB): 50%-recovery-at-3 σ threshold at $A = 0.75\%$.

To make the mapping from estimator to scientific claim explicit — and to forestall any reading of the multi-null residual structure as a “forest” of competing detection significances — Table I is a decision tree: each *scientific claim* the paper makes is tied to exactly one estimator class, its analysis sample, its null, and its role. Only the two rows marked PRIMARY carry cosmological weight; every harmonic-channel σ is a systematics diagnostic and supports no independent detection claim. This is the single load-bearing map for the entire Results section: a reader who keeps only Table I retains the complete logical chain from data to the null conclusion.

Table II consolidates per-estimator N_{spiral} , f_{sky} , mask, null type, and reported σ . The spread of reported significances (from $+0.41\sigma$ to $+7.93\sigma$) across these rows is *not* a set of conflicting measurements of one quantity requiring statistical reconciliation: each row is a distinct *observable* (real-space dipole vs. single-mode pseudo- C_ℓ vs. hemisphere max-statistic vs. template-fit amplitude) evaluated on a distinct sample against a distinct null, so the values are not commensurable as detection significances and no joint likelihood over them is defined or claimed. The scientific verdict rests solely on the two rows tagged PRIMARY — the real-space HC dipole (consistent with null) and the block-bootstrap WLS template fit (a clean 1.7% dipole disfavored); every DIAGNOSTIC row is, by design, a systematics characterization whose elevated σ is attributed (Sec. IV D, Appendix D) rather than interpreted as signal. The apparent tension is therefore between *systematics diagnostics and the pri-*

TABLE I. **Estimator decision tree — which estimator supports which scientific claim.** Each row is a distinct claim made in this paper, tied to its estimator (cross-referenced to the hierarchy rows above and to Table II), the analysis sample with its explicit spiral count N , the null procedure, and its role. PRIMARY rows are the only cosmological detection/exclusion claims; DIAGNOSTIC rows characterize systematics and assert *no* independent signal; the CALIBRATION row sets the sensitivity/falsification scale. Within a role the σ/z values are internally meaningful; *across* rows they are not comparable as detection significances (each is against its own null, listed in the “Null” column).

Scientific claim	Estimator	Sample (N_{spiral})	Null	Role
Real-space chirality dipole is consistent with zero	(i) HC real-space dipole	HC, $p_{\text{eq}} > 0.6$ (949,584)	isotropic pixel-permutation (10^4)	PRIMARY
A clean cosmological 1.7% dipole is disfavored	(ii) block-bootstrap WLS template fit	full Catalog C (3,201,160)	block-bootstrap (10^3)	PRIMARY
Harmonic $\ell=1$ residuals are systematics (monopole-mask leakage + coherent depth/morphology), <i>not</i> a cosmological signal	(iii,iv) canonical/apodized MASTER; (vi) monopole-only generative null	full Catalog C (3,201,160)	label-shuffle / depth-stratified / monopole-only	DIAGNOSTIC
No directional hemispheric asymmetry survives look-elsewhere control	(v) hemisphere max-asymmetry	full Catalog C (3,201,160)	max-statistic MC (p_{LEE})	DIAGNOSTIC
Sensitivity / falsification scale of the primary real-space estimator	(vii) injection-recovery	HC, $p_{\text{eq}} > 0.6$ (949,584)	per-pixel shuffle	CALIBRATION

mary null, and is resolved in the systematics direction by the eight-anchor battery, not left as an unmodeled statistical inconsistency.

C. Model Architecture

The classifier consists of a ViT-Small encoder [12] (`vit_small_patch16_224`, ImageNet-pretrained) with the last 6 of 12 transformer blocks fine-tuned, followed by a custom classification head:

$$\begin{aligned} \text{LayerNorm} &\rightarrow 384 \rightarrow 512 \text{ (GELU, } d=0.3) \\ &\rightarrow 512 \rightarrow 256 \text{ (GELU, } d=0.2) \rightarrow 256 \rightarrow 3 \text{ (softmax)}. \end{aligned} \quad (1)$$

The three-class output (P_{CW} , P_{CCW} , P_{NS}) is essential for full-survey deployment: applying a binary classifier to data where $\sim 70\%$ of objects are elliptical or irregular produces a catalog dominated by noise. Full architecture and training details are in Appendix B.

D. Test-Time Equivariant Averaging

At inference, each galaxy is classified on both the original image and its horizontal reflection. The equivariant probability is:

$$\begin{aligned} P_{\text{CW}}^{\text{eq}} &= \frac{1}{2}(P_{\text{CW}}^{\text{orig}} + P_{\text{CCW}}^{\text{flip}}), \\ P_{\text{CCW}}^{\text{eq}} &= \frac{1}{2}(P_{\text{CCW}}^{\text{orig}} + P_{\text{CW}}^{\text{flip}}), \\ P_{\text{NS}}^{\text{eq}} &= \frac{1}{2}(P_{\text{NS}}^{\text{orig}} + P_{\text{NS}}^{\text{flip}}). \end{aligned} \quad (2)$$

This procedure enforces flip-equivariance of the output protocol (flip-swap correlation = 1.000). We restrict to 2-fold TTA (original + horizontal flip) rather than the full D_4 group because mirrors flip chirality by definition, whereas in-plane rotations do not change chirality; rotation-TTA probes classifier non-equivariance rather than the chirality assignment itself. Because Eq. (2) enforces flip-equivariance at inference by construction, the flip-swap consistency test T1 (Appendix B) is a *protocol implementation check* that guards against code defects in the TTA averaging, not an independent statistical test of the learned model’s equivariance; the pre-TTA behavior of the underlying network is documented separately by the raw single-pass Catalog A tier (Table III). A direct D_4 -TTA hold-out on two independent $\sim 2,000$ -galaxy subsamples confirms the mean per-galaxy P_{CW} is stable under Z_2 and D_4 to within $|\Delta\langle p_{\text{CW}} \rangle| < 0.0016$; per-galaxy argmax labels flip in 21.4% of cases between Z_2 and D_4 on borderline galaxies with $P_{\text{CW}} \approx P_{\text{CCW}} \approx 0.4$. This D_4 hold-out is a classifier-stability check (comparing softmax outputs under two augmentation groups), not a spatial-null or isotropy test; spatial-null calibration is performed via the isotropic (pixel-)permutation nulls of Sec. IV C. Full details in Appendix B.

E. Catalog Tiers

The pipeline produces three tiers: **Catalog A** (raw, single-pass softmax); **Catalog B** (Platt-calibrated, +0.4% excess); **Catalog C** (equivariant production, 2-fold flip TTA). **Catalog C is the recommended tier**

TABLE II. Primary-estimator summary. $N_{\text{catalog spiral}}$ is the underlying Catalog C spiral count (CW+CCW only). $N_{\text{map weighted}} = \sum_{p \in \text{mask}} W_p$ where $W_p = N_{\text{all}}^{(p)}$ is the total classified-galaxy count in pixel p (CW+CCW+NS), used as a survey-depth weight in the NaMaster field object. $N_{\text{map weighted}}$ exceeds $N_{\text{catalog spiral}}$ because W_p includes non-spiral galaxies ($\sim 62\%$ of the catalog); each galaxy is counted once. Row (iv) is the apodized-footprint MASTER diagnostic ($N_{\text{all}} \geq 1$ mask, C^2 2° apodization); its two σ values are against the global per-galaxy label-shuffle and depth-stratified nulls respectively (both systematics-attributed; Appendices A, D). *The σ values in different rows are computed against different null procedures (column “Null”) and are not directly comparable across rows as detection significances.* Row (v) reports the post-look-elsewhere-corrected significance; the raw direct-MC value is $p_{\text{LEE}} \leq 10^{-4}$ against the random-label max-statistic null, which *already incorporates* the look-elsewhere scan and is the principled directional look-elsewhere control (Appendix C); the rejection is systematics-attributed. Row (vii) uses the HC-broad subsample ($p_{\text{eq}} > 0.6$; Sec. VIB). A “—” in $N_{\text{map weighted}}$ marks rows where the NaMaster weight construction is not invoked.

Estimator	$N_{\text{catalog spiral}}$	$N_{\text{map weighted}}$	f_{sky}	Mask	Null	Reported statistic
(i) real-space dipole (HC)	949,584	—	0.4801 ^a	canonical	pix-perm. (10^4)	+0.41 (moment z)
(ii) WLS template excl.	3,201,160	—	0.49005	canonical	block-boot. (10^3)	$z \approx -18$
(iii) canonical MASTER (diag.)	3,201,160	—	0.49005	canonical	pp-shuffle	+3.64
(iv) apod. MASTER (diag.)	3,201,160	8,474,531	0.494 ^b	$N_{\text{all}} \geq 1$ apod.	pp-sh./d-str.	+7.28/+7.13
(v) hemisphere LEE (MC)	3,201,160	—	0.49005	canonical	max-stat MC	$p_{\text{LEE}} \leq 10^{-4}$ (syst.-attr.)
(vi) monopole+mask null	3,201,160	—	0.49005	canonical	monopole-only	+1.69
(vii) injection floor	949,584 HC	—	—	—	pp-shuffle	50%-rec- 3σ , $A=0.75\%$

^a $f_{\text{sky}} = 0.4801$ is the canonical $N_{\text{spiral}}(p) \geq 10$ mask re-evaluated on the high-confidence subsample: fewer HC spirals populate fewer pixels at the ≥ 10 threshold (23,600 of 49,152 pixels), slightly shrinking the full-catalog canonical mask ($f_{\text{sky}} = 0.49005$, Appendix A Table IX). Artifact: [pipelines/p2_chirality/outputs/dipole/catalog_c_summary.json](#).

^b Geometric $N_{\text{all}} \geq 1$ footprint pixel fraction; the corresponding weighted/apodized *effective* sky fraction for this row’s $W_p = N_{\text{all}}$, C^2 2° configuration is $f_{\text{sky}}^{\text{eff}} = 0.452$ (Appendix A, Table IX).

for all cosmological parity analyses. All three tiers share 8,474,531 rows in Apache Parquet format.

IV. RESULTS

Significance conventions. This paper reports significance in standard-deviation units (σ) throughout, but values from distinct null procedures are *not* directly comparable. Section identifiers specify the null for each result. Empirical rank p -values are one-sided unless explicitly labeled two-sided.

A. Catalog Statistics

The final catalog contains 8,474,531 galaxies (157 of 8,474,688 failed quality checks). Catalog C (equivariant): CW 1,592,107 (18.78%), CCW 1,609,053 (18.99%), NS/edge-on 5,273,371 (62.23%); spiral total $N_{\text{spiral}} = 3,201,160$ (37.77%) (percentages rounded to maintain sum-to-one consistency at the second decimal; exact values: CW 18.787%, CCW 18.987%, NS 62.226%, spiral 37.774%; the integer counts are exact). The spiral fraction is consistent with magnitude-limited survey expectations. Mean classification confidence is 0.951, median 0.9997. We caution that these max-class probabilities are *not probabilistically calibrated*: the catalog-wide mean confidence (0.951) far exceeds the independent GZ1 three-class accuracy (58.7%; spiral-chirality 69.91%, Appendix B), i.e. the classifier is strongly overconfident relative to external truth. The p_{eq} “high-confidence” cuts

used throughout are therefore monotone sample-selection thresholds (rankings by classifier confidence), not statements that a selected label is correct with probability p_{eq} ; the injection-recovery floors of Sec. VIB are defined operationally on the threshold-selected subsamples and do not assume calibration. Crucially, the primary dipole estimator consumes *hard argmax* CW/CCW counts per pixel, not the confidence scores themselves, so probabilistic miscalibration cannot bias the dipole *amplitude or direction*: overconfidence affects only which galaxies enter a given p_{eq} -selected subsample, and the confidence-cut sweep of Sec. IV C ($p_{\text{eq}} \in \{0, 0.4, 0.5, 0.6, 0.7, 0.8\}$) directly maps that selection dependence (the null verdict is stable across the entire high-confidence regime). What *is* propagated into the science is the external GZ1 chirality accuracy (69.91%), which enters the conservative dilution factor $g = 2a - 1$ folded into the empirical floors (Sec. II, VIB) — i.e. the load-bearing validation against independent human labels is the GZ1 cross-match, not the internal softmax confidence.

B. Global CW Fraction

The Catalog C residual (-9.5σ from 0.5000, Table III) is spatially uniform across 7 equatorial coordinate slabs and does not produce a dipole. This monopole offset is a classifier artifact, not a physical signal; three candidate mechanisms are (1) GZ1 training-label CW excess, (2) residual orientation-dependent bias not corrected by 2-fold TTA, and (3) photometric asymmetry in DESI Legacy imaging. The $2.98\times$ asymmetry-

Equivariant Averaging: Original vs. Flipped Predictions

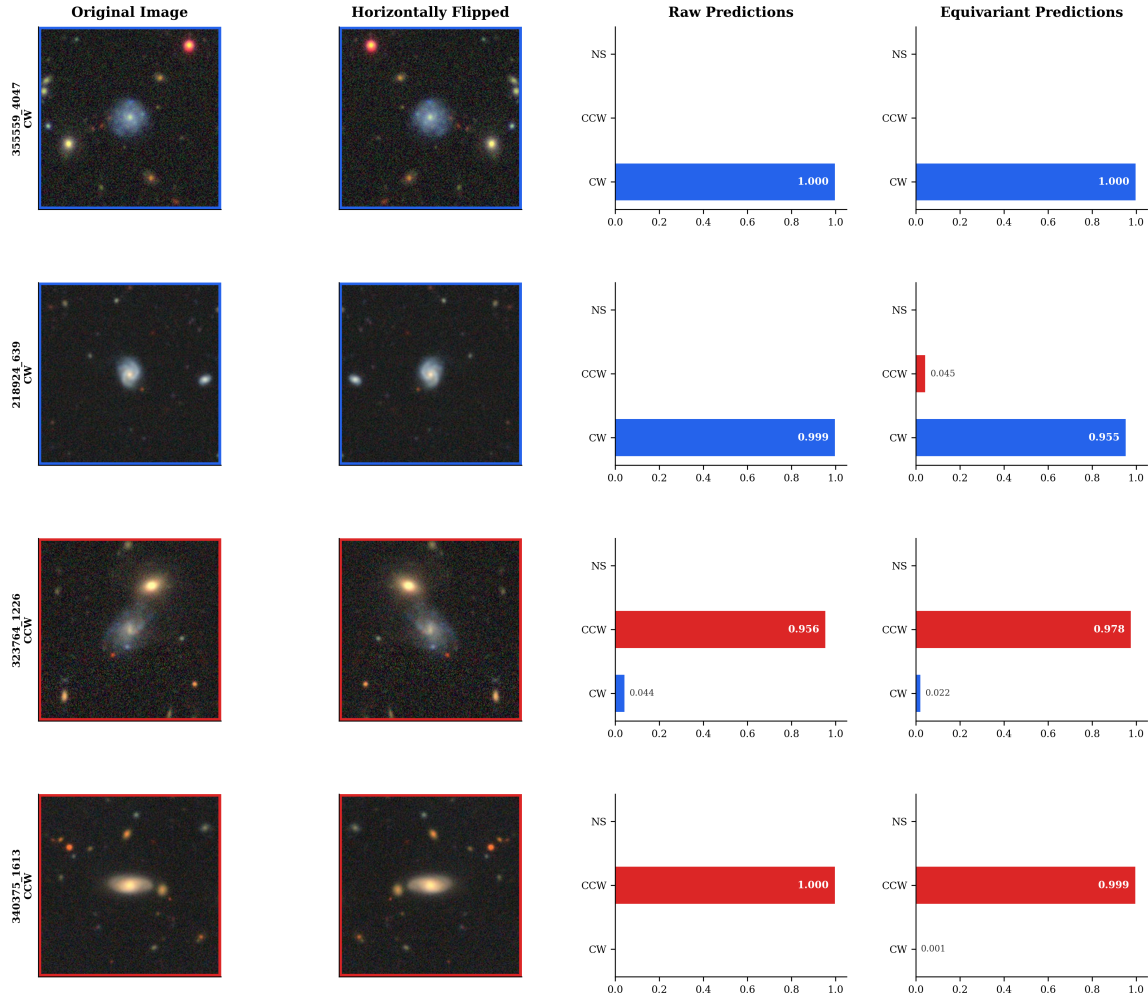


FIG. 2. **Equivariant test-time averaging (TTA)**. Representative Z_2 production TTA examples (original + horizontal flip); D_4 validation (four rotations \times two reflections) in Appendix B. *Production* inference (Catalog C) uses 2-fold Z_2 TTA — original + horizontal flip only (§III D). Flips swap the CW \leftrightarrow CCW class labels by construction. Output probabilities are averaged after the swap, yielding a strictly flip-equivariant CW/CCW classifier with flip-swap correlation = 1.000 by construction. This averaging is the key methodology distinction between Catalog A (raw), Catalog B (Platt-calibrated), and Catalog C (equivariant); the global chirality asymmetry $(N_{\text{CW}} - N_{\text{CCW}})/N_{\text{spiral}}$ shifts from +1.576% (A) to -0.529% (C), i.e. $+0.788\%$ to -0.265% in f_{CW} -deviation units, dominated by this step (Table III). Unit reminder: asymmetry- A values are exactly twice the f_{CW} -deviation values, $A = 2(f_{\text{CW}} - \frac{1}{2})$ (Sec. IV C); Table III quotes the f_{CW} -deviation convention.

suppression factor from raw $+1.576\%$ to equivariant -0.529% (asymmetry- A units, $A = 2(f_{\text{CW}} - \frac{1}{2})$; equivalently $+0.788\% \rightarrow -0.265\%$ in f_{CW} -deviation units, Table III) demonstrates the dominance of the equivariant TTA processing. Crucially, the slab statistics support this quantitatively: in 7 equal-spiral-count declination slabs ($N = 457,308$ – $457,309$ spirals each; per-slab binomial $\sigma = 7.4 \times 10^{-4}$), the per-slab f_{CW} spans 0.49537 – 0.49890 , i.e. deviations from 0.5 of -0.110% to -0.463% , all within 0.5% of 50/50; an equal-count RA partition gives a compatible span (-0.060% to -0.501%). An equal-*area* partition (8 declination bands of equal in-mask pixel count on the canonical

mask) gives the same verdict: per-band f_{CW} deviations from 0.5 reach at most 0.49% on the full spiral sample (max $|z| = 2.9$ vs. the global rate) and 0.56% on the HC subsample (max $|z| = 1.4$), comparable to the equal-count maximum of 0.46% (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json](#)). The slab-to-slab scatter about the global $f_{\text{CW}} = 0.49735$ is $\lesssim 2.7\sigma$ per slab (the extremal slab $f_{\text{CW}} = 0.49537$ gives $(0.49735 - 0.49537)/(7.4 \times 10^{-4}) = 2.7\sigma$ from the per-slab values and binomial σ above; artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json](#)), consis-

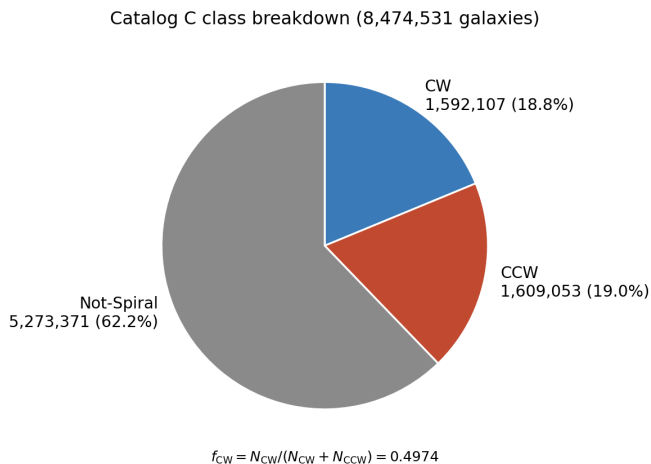


FIG. 3. **Catalog C composition.** Of the 8,474,531 galaxies retained after image-quality QA, the equivariant TTA classifier (§III D) assigns $N_{CW} = 1,592,107$, $N_{CCW} = 1,609,053$, and $N_{NS} = 5,273,371$ (non-spiral / edge-on / morphologically indeterminate). The spiral sub-catalog $N_{\text{spiral}} = N_{CW} + N_{CCW} = 3,201,160$ is the analysis target for all chirality statistics below (Table III *et seq.*).

TABLE III. Global CW fraction across catalog tiers. Uncertainties are 1σ binomial, $\sigma = \sqrt{f(1-f)/N_{\text{spiral}}}$, with per-tier spiral counts ($N_{\text{spiral}}^A = 3,321,795$; $N_{\text{spiral}}^C = 3,201,160$); parentheses give the 1σ uncertainty on the trailing digits ($0.507879(274) \equiv 0.507879 \pm 0.000274$). The Excess column is the deviation $f_{CW} - 0.5$ in percent (f_{CW} units; multiply by 2 for asymmetry- A units). Dev. is the *signed* $(f_{CW} - 0.5)/\sigma$ computed from the unrounded fraction. The Catalog-B row derives from the Platt-calibrated fraction; its deviation is computed from the unrounded calibrated fraction. Percentages in this table (and in the catalog-composition counts cited in Sec. II) are rounded to maintain sum-to-one consistency at the second decimal; the integer counts are exact.

Tier	cw/(cw + ccw)	Excess (%)	Dev. (σ)
A (raw)	0.507879(274)	+0.788	+28.72
B (calibrated)	0.50400(27)	+0.4	+14.6
C (equivariant)	0.497353(279)	-0.265	-9.47

tent with the coherent low- ℓ systematic structure dis-positioned in Appendix D rather than a dipole-aligned gradient. Independently of any slab partition, a direct generative test confirms that a constant monopole cannot bias the uniform-weight real-space dipole estimator (the constant template is absorbed by the fitted monopole term): binomial per-pixel realizations on the canonical mask drawn at $p = f_{CW}^{\text{global}}$ versus $p = 0.5$ yield statistically identical dipole-amplitude null distributions (means 1.957×10^{-3} vs. 1.935×10^{-3} , a 0.39σ shift in the standard error of the difference; $N = 500$ each; artifact [pipelines/p2_chirality/outputs/canonical_provenance/c11_meta_m4_slab_stats.json](#)). This generative test probes an *additive constant* monopole only; multiplicative depth- or morphology-coupled mod-

ulations of the monopole are not probed by it and are addressed instead by the template-regression and cross-spectrum diagnostics of Appendix D. The sub-percent sensitivity claim should therefore be interpreted as a floor on the physical isotropy-breaking dipole signal. *Implications for $\ell = 0$ parity searches.* The morphological handedness monopole is itself the parity-odd channel (unlike the parity-even $\ell = 1$ dipole; Sec. VI C), so a naive reading of the -9.47σ deviation as cosmic parity violation would be the single largest false-positive risk in any $\ell = 0$ chirality-parity study on such a catalog. Our analysis shows this deviation is a spatially-uniform classifier artifact ($\lesssim 0.5\%$ per-slab scatter, three candidate instrumental/training mechanisms above), not a physical signal; the concrete lesson for future $\ell = 0$ parity searches is therefore that the released catalog labels must be locally monopole-renormalized (per-region CW-fraction subtraction) before any global parity-odd statistic is formed, and that any claimed $\ell = 0$ parity detection at or below the $\approx 0.26\%$ artifact level (9.5σ in raw counts) is indistinguishable from this classifier monopole without an independent, handedness-symmetric labeling pipeline (Sec. E, Data Availability).

C. Dipole Analysis

We pixelize the sky at HEALPix resolution NSIDE = 64 (49,152 pixels, ~ 0.84 deg² per pixel). In each pixel p containing $N_{\text{spiral}}(p) \geq 10$ spiral galaxies (this $N_{\text{spiral}}(p) \geq 10$ per-pixel cut is the canonical mask, $f_{\text{sky}} = 0.49005$, and is the single mask definition referenced by every figure and table for every quoted number in this paper unless explicitly noted; Appendix A), we compute the asymmetry

$$A_p = \frac{N_{CW}^{(p)} - N_{CCW}^{(p)}}{N_{CW}^{(p)} + N_{CCW}^{(p)}}. \quad (3)$$

This is the single canonical chirality-field definition used throughout (spirals-only denominator). Note the unit convention $A_p = 2(f_{CW,p} - \frac{1}{2})$: amplitudes quoted in A_p units are exactly twice the corresponding f_{CW} -deviation amplitudes.

a. Simple dipole. The real-space estimator fits the monopole-plus-dipole model $A_p = m + \mathbf{a} \cdot \hat{n}_p$ to the in-mask pixel map by least squares with uniform pixel weighting (`healpy.fit_dipole`); the dipole amplitude is $A_{\text{dip}} = |\mathbf{a}|$. The null distribution is built from $N_{\text{MC}} = 10,000$ isotropic realizations in which the per-pixel asymmetry values A_p are randomly permuted across the in-mask pixels (destroying any coherent dipole while preserving the one-point distribution) and the fit repeated; the quoted p is the *one-sided empirical rank* of the observed amplitude in this null (the two-sided equivalent is 0.62). Because A_{dip} is positive-definite, the (z, p) pair does not follow the Gaussian $z \rightarrow p$ mapping ($z = +0.41$ is the moment-ratio against the null mean

Galaxy Chirality Asymmetry Map (8.47M galaxies, equivariant)

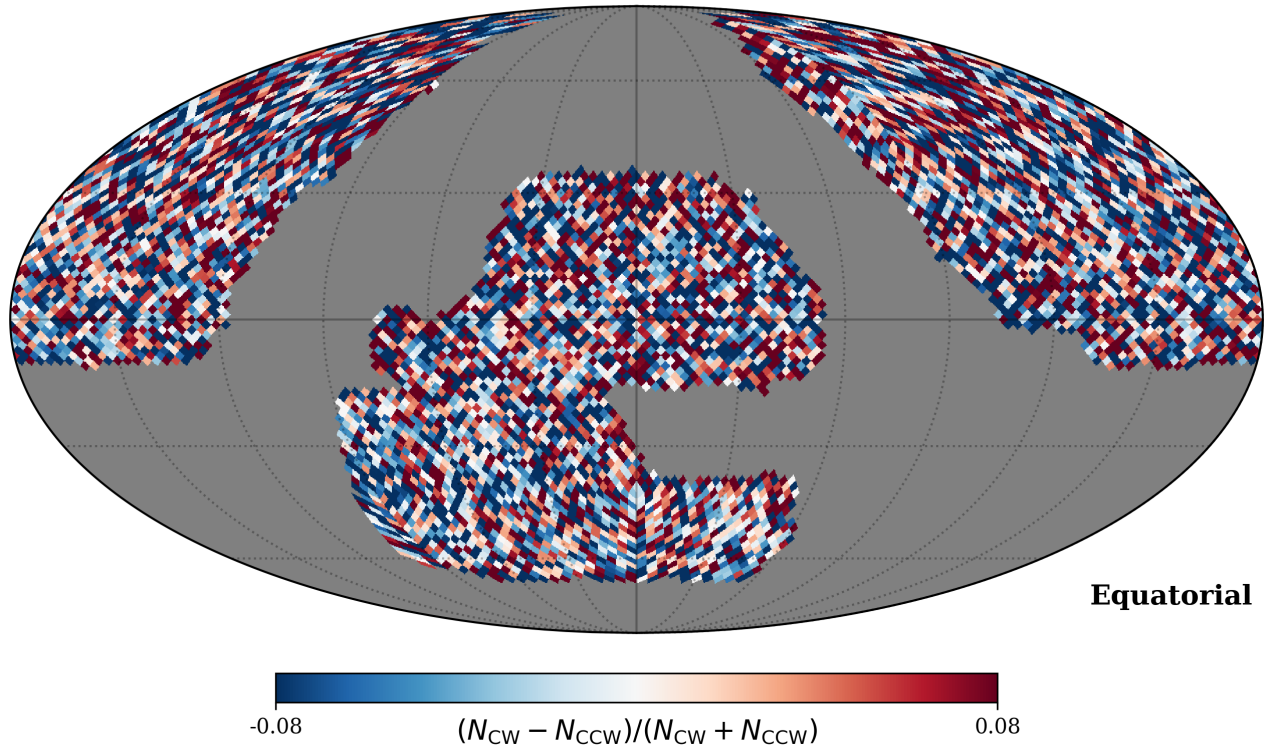


FIG. 4. **Equivariant (Catalog C) chirality asymmetry map of the 8.47 M-galaxy catalog** (Mollweide projection, equatorial coordinates; per-pixel asymmetry $A_p = (N_{CW} - N_{CCW})/(N_{CW} + N_{CCW}) = 2(f_{CW,p} - \frac{1}{2})$ at HEALPix NSIDE=64, color scale $[-0.08, +0.08]$). The DESI Legacy Imaging footprint covers $f_{\text{sky}} = 0.49005$ of the sky in the canonical mask ($N_{\text{spiral}}(p) \geq 10$ per pixel; Sec. IV C); the $N_{\text{all}} \geq 1$ analysis footprint ($f_{\text{sky}} = 0.494$) is used for the apodized MASTER diagnostic. Spatial uniformity of the per-pixel CW fraction is verified across 7 equatorial coordinate slabs (§IV B); the canonical-mask $\ell=1$ residual is analyzed in Sec. IV C–IV D. (σ values across this paper’s panels arise from distinct null procedures; see Sec. III A.)

and width; $p = 0.31$ is the rank). The primary fit uses the high-confidence Catalog C selection of the generator script (`pipelines/p2_chirality/run_dipole_catalog_c.py`; equivariant spirals with winning-class confidence > 0.6 ; $N_{\text{HC}} = 949,584$). *We state the pre-registration rationale for this cut here, before quoting the dipole result below, precisely so it cannot be read as a post-hoc justification*: this 0.6 threshold is the operational definition the generator has used throughout and is *not* tuned post-hoc against the dipole outcome. *Pre-registration and purity-completeness rationale for the 0.6 cut*. The $p_{\text{eq}} > 0.6$ primary selection was fixed *a priori* in the generator script (`pipelines/p2_chirality/run_dipole_catalog_c.py`) before the dipole was evaluated, and 0.6 is the natural purity-completeness balance point for this catalog, not an arbitrary choice between 0.5 and 0.7. On the uncalibrated three-class winning-score p_{eq} , a cut at 0.5 retains essentially the full argmax-classified spiral sample (it is the argmax boundary itself and so admits the entire depth-correlated low-confidence tail — exactly the population carrying the $z \approx 4.0\text{--}4.3$ systematic excess documented below), sacrificing purity for no completeness gain of scientific value; a cut at

0.7 or 0.8 discards $\sim 50\%/ \sim 75\%$ of the high-confidence spirals (N falls from 949,584 at 0.6 to 471,049 at the stricter 0.9 subsample), inflating the sensitivity floor (A_{50} rises with $N^{-1/2}$, Sec. VIB) without changing the null verdict. The 0.6 cut is thus the lowest threshold that already excludes the depth-correlated low-confidence tail (the transition localized at $p_{\text{eq}} \leq 0.6$ by the sweep below) while retaining maximal completeness for the tightest achievable floor — the standard purity-maximizing-subject-to-completeness criterion. Critically, this choice is *not* a researcher degree of freedom regardless of the rationale: the full confidence-cut sweep below ($p_{\text{eq}} \in \{0, 0.4, 0.5, 0.6, 0.7, 0.8\}$) shows the null verdict is robust across the entire high-confidence regime ($z = +0.41, +1.14, +0.51$ at 0.6, 0.7, 0.8 — all consistent with the null at $|z| < 1.2$, the residual spread being statistical), so the scientific conclusion is invariant to the exact cut and no forking-paths concern applies. The GZ1 validation cross-match makes the tradeoff concrete with a few numbers: on the 234,282-galaxy disjoint GZ1 cross-match (Appendix B, Table XI), the equivariant classifier’s chirality purity (predicted-CW that are truly CW, and symmetrically for CCW) is

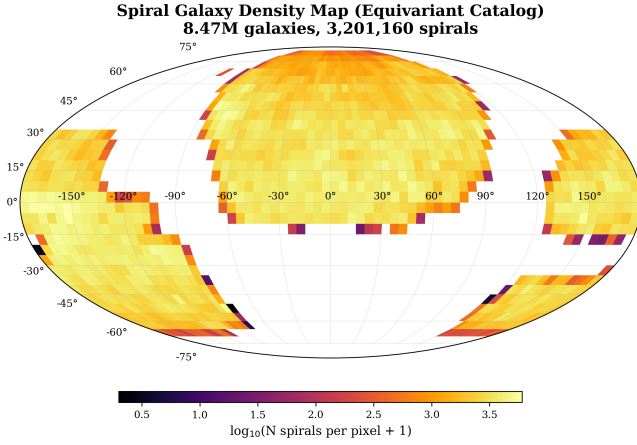


FIG. 5. Sky density of the 3,201,160 classified spirals (CW + CCW combined, NSIDE=64 Mollweide). Per-pixel spiral counts scale with the DESI Legacy Imaging Surveys depth and exposure pattern; the canonical mask used for the primary $\ell = 1$ analysis (§IV C) requires $N_{\text{spiral}}(p) \geq 10$ per pixel. Spatial inhomogeneity at this scale is the leakage channel quantified in §IV D.

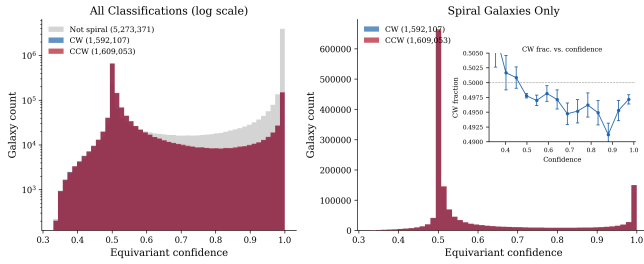


FIG. 6. Distribution of maximum-class confidence $\max(P_{\text{CW}}, P_{\text{CCW}}, P_{\text{NS}})$ for all 8,474,531 galaxies. Strongly bimodal: 73.6% at $\max p \geq 0.9$ (high-confidence labels) + a long tail of indeterminate cases ($\max p < 0.5$, dominated by NS/edge-on systems). The high-confidence (HC) cuts at $p_{\text{eq}} > 0.6$ ($N = 949,584$) and $p_{\text{eq}} > 0.8$ ($N = 624,660$) used in the systematics cross-checks (§E) are indicated. (σ values across this paper’s estimators arise from distinct null procedures; see Sec. III A.)

$\approx 70\%$ integrated over all confidences (70.4% CW, 69.4% CCW), and rises monotonically with the confidence score p_{eq} ; imposing $p_{\text{eq}} > 0.6$ retains $N_{\text{HC}} = 949,584$ of the 3,201,160 classified spirals ($\approx 30\%$ completeness of the full spiral sample) while excluding the $p_{\text{eq}} \leq 0.6$ low-confidence tail that carries the $z \approx 4.0$ – 4.3 depth-correlated systematic excess; tightening to $p_{\text{eq}} > 0.9$ raises purity further but halves completeness again ($N = 471,049$, $\approx 15\%$), inflating the A_{50} floor by $\sqrt{949,584/471,049} \approx 1.42\times$ for no change in the null verdict. The 0.6 cut is thus the completeness-maximizing choice subject to excluding the systematic-dominated tail. (A formal pre-unblinding purity-completeness curve derived on the GZ1 validation set — rather than the a-posteriori sweep-stability argument and

integrated purity numbers used here — would further document the choice and is noted as a straightforward extension.) The fitted dipole has amplitude 4.4×10^{-3} toward $(l, b) = (293^\circ, 12^\circ)$ with significance 0.41σ (rank- $p = 0.31$), fully consistent with the null hypothesis; at this significance the dipole axis is unconstrained (under the null the recovered direction is effectively a random draw, so we attach no uncertainty contour to the quoted (l, b)). Three robustness checks on this primary: (i) replacing the pixel-permutation null with a *per-galaxy label-shuffle* null — which preserves the per-pixel counts $N_{\text{spiral}}(p)$ and their Poisson noise geometry — leaves the verdict unchanged (0.58σ , rank- $p = 0.26$, same generator; an independent implementation gives $z = 0.70$, rank- $p = 0.23$, artifact [pipelines/p2_chirality/outputs/canonical_provenance/c11b_hc_dipole_nulls.json](#)); (ii) an independent uniform-weight least-squares re-implementation of the fit reproduces the pixel-permutation verdict ($z = 0.55$, rank- $p = 0.27$, same artifact); (iii) a 2×3 robustness panel crossing the fit weighting (uniform vs. $N_{\text{spiral}}(p)$ -weighted) with the mask threshold ($N_{\text{spiral}}(p) \geq 10, 20, 50$) leaves every cell consistent with the null ($|z| \leq 0.8$, rank- $p \geq 0.20$; 2000-permutation pixel nulls per cell, artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json](#)). The regenerated 10^4 -permutation null array also yields a formal null-quantile benchmark: defining $A_{95, \text{nq}}$ as the 95th percentile of the pixel-permutation null amplitude distribution (“null-quantile”; *not* a signal-injected limit and carrying no frequentist coverage guarantee), i.e. the smallest amplitude with $P(A_{\text{null}} \geq A_{95, \text{nq}}) = 0.05$ (an estimator-level rank construction; the conservative companion $\max(A_{\text{obs}}, A_{95, \text{nq}})$ coincides with it since $A_{\text{obs}} < A_{95, \text{nq}}$, and is a descriptive estimator-level bound used in no scientific conclusion), gives $A_{95, \text{nq}} = 6.8 \times 10^{-3}$ in A_p units — for a pure dipole the A_p -dipole amplitude equals the full-amplitude A of the injection convention ($p_{\text{CW}} = p_0 + (A/2) \cos \theta \Rightarrow A_p$ -dipole amplitude = A), so this is numerically on the same scale as the $A_{50} \approx 0.75\%$ injection floor while remaining a different object (null {50, 68, 90, 95, 99}% quantiles = $\{3.5, 4.4, 6.0, 6.8, 8.4\} \times 10^{-3}$; same artifact). *Sensitivity to the confidence threshold*: dropping the confidence threshold entirely (all 3,200,420 in-mask equivariant spirals; the remaining 740 of the catalog’s 3,201,160 spirals lie in pixels below the $N_{\text{spiral}}(p) \geq 10$ canonical-mask threshold) yields $A_p = 0.0057$ (= 0.57%) dipole at $z \approx 4.2$ – 4.4 under both null constructions (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c11_meta_e1_e2_realspace_nulls.json](#)); this unthesholded excess is attributed to residual depth-correlated classifier bias concentrated in the low-confidence tail — the same systematic family dispositioned for the harmonic-channel residuals (Sec. IV D, Appendix D) — and its amplitude sits below the

HC-broad $A_{50} \approx 0.75\%$ falsification-criterion floor; it is reported here as a systematics-sensitivity diagnostic, not a detection. (An empirical injection floor measured directly on the unthresholded full-sample estimator ($N = 3,201,160$ spirals, all $N_{\text{spiral}}(p) > 0$ pixels; same WLS estimator and per-pixel binomial null, area-uniform axis draws, $N_{\text{MC,inj}} = 200$ per amplitude) gives $A_{50} \approx 0.36\%$ and $A_{95} \approx 0.63\%$ (log-interpolated recovery-fraction crossings; smallest tested amplitudes crossing 50%/95%: 0.5%/0.75%; artifact [pipelines/p2_chirality/outputs/canonical_provenance/c16_r24conf_pod_batch.json](#)). The $A_p = 0.0057$ ($= 0.57\%$) unthresholded excess therefore lies between the full-sample A_{50} and A_{95} — consistent with its $z \approx 4.2$ – 4.4 visibility — and the systematic attribution rests on the confidence-cut sweep below, not on a sub-floor amplitude argument.) The confidence threshold is a monotone selection cut on uncalibrated scores (see the calibration caveat in Sec. II), so the primary is defined at the threshold the generator script has used throughout. A full confidence-cut sweep ($p_{\text{eq}} \in \{0, 0.4, 0.5, 0.6, 0.7, 0.8\}$; 2000-permutation pixel nulls each) localizes the transition: $z = +4.3, +4.1, +4.0$ at cuts 0, 0.4, 0.5, collapsing to $z = +0.41, +1.14, +0.51$ at 0.6, 0.7, 0.8, confirming the excess is confined to the $p_{\text{eq}} \leq 0.6$ low-confidence tail (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json](#)). In contrast, Catalog A (raw) shows a 2.31σ real-space dipole and a $+6.48\sigma$ pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ ¹ at the lowest multipole ($\ell=1$; a single decoupled mode, not a binned bandpower)—both entirely artifacts of the model’s residual CW bias spatially modulated by non-uniform survey depth, and both collapsed to null by two complementary reductions (equivariant TTA averaging in real space and MASTER mode-coupling deconvolution in spherical-harmonic space).

b. Angular power spectrum. We perform MASTER mode-coupling deconvolution using NAMASTER [32] at $N_{\text{side}} = 64$ on the real analysis footprint ($N_{\text{all}} \geq 1$; 24,297 pixels, $f_{\text{sky}} = 0.494$, C^2 2° apodization, depth weight W_p). The MASTER-deconvolved single-mode $\ell=1$ value (computed on the monopole-subtracted A_p field; declared data vector, Appendix A) is $C_1^{\text{meas}} = 2.348 \times 10^{-5}$ against a 500-MC per-galaxy label-shuffle null with mean 1.71×10^{-6} and $\sigma_{\text{null}} = 2.99 \times 10^{-6}$, i.e. $+7.28\sigma$ for $W_p = N_{\text{all}}$ ($+9.78\sigma$ for $W_p = N_{\text{spiral}}$); the 10^4 -permutation recompute (Table IV) confirms this channel at $z = +7.31$ with empirical rank $p = 6.0 \times 10^{-4}$ (one-sided, computed as $(k+1)/(N+1)$ with $k=5$ of $N=10^4$ null draws exceeding the data; Table IV caption), and the

¹ Catalog-A pre-MASTER $\ell=1$: single-mode pseudo- $C_\ell^{(\ell=1)}$ on the canonical mask ($N_{\text{spiral}}(p) \geq 10$, $f_{\text{sky}} = 0.49005$) without MASTER deconvolution, per-pixel label-shuffle null (same estimator family as the canonical Catalog-C $+3.64\sigma$ result; Sec. IV D).

Gaussian-equivalent σ is quoted from the null moments. A depth-stratified null (labels permuted within 10 $N_{\text{all}}(p)$ deciles; this preserves the marginal depth distribution but not joint spatial–depth structure, so it bounds only depth-sampling effects) leaves the excess essentially unchanged ($+7.13\sigma$ / $+9.06\sigma$), excluding pixel-depth sampling alone as the driver. A weight-map sweep gives the same verdict: the excess persists under all three weightings ($W_p = N_{\text{all}} + 6.9\sigma$, $W_p = N_{\text{spiral}} + 8.5\sigma$, uniform binary $+6.7\sigma$; independent 500-MC null streams, artifact c9c; in each variant the mask-mean subtraction uses the same variant weight map, so the field-consistent N_{spiral} -weighted subtraction is included in the sweep), so it is not an artifact of the depth-weight choice, although its magnitude is weight-dependent at the $\pm 1\sigma$ level. This channel is therefore a *systematics diagnostic* on the patchy weighted footprint — the same coherent low- ℓ family as the canonical-mask residual (Sec. IV D, Appendix D) — and is not used as a cosmological null; the cosmological statements of this paper rest on the real-space dipole and the template-fit exclusion (Appendix D). NaMaster configuration details and the canonical mask-declaration + depth-stratification audit are in Appendix A. We emphasize that rows (i) and (iv) of Table II are not on the same statistical footing — different fields, masks, weights, and null procedures — so the apparent $+0.41\sigma$ vs. $+7.28\sigma$ gap is not a $17\times$ discrepancy on a common axis; the harmonic-completeness check (Sec. VII, artifact c9b) bounds what a clean Shamir-class real-space dipole would have produced in the MASTER channel — it would register at $z \approx 68$ – 218 there, versus the observed $+7.28\sigma$ — but it does not establish statistical consistency of the two estimators on a common axis.

D. Monopole+Mask Leakage Generative Null

The canonical-mask direct-MC $\ell=1$ value of $+3.64\sigma$ (per-pixel label-shuffle null) and the local hemisphere maximum of 3.05σ (label-shuffle null, 648-direction scan; Appendix C) are candidate manifestations of mask-geometric leakage of the global 9.5σ monopole. We formalize this with a generative null: $N = 500$ realizations in which the per-pixel CW count is drawn from $\text{Binomial}(N_{\text{spiral}}(p), p_{\text{CW}}^{\text{global}})$ on the exact canonical mask, with no injected dipole.²

² Here $N_{\text{spiral}}(p) \equiv N_{\text{CW}}(p) + N_{\text{CCW}}(p)$ is the per-pixel *spiral* count, not the all-galaxy per-pixel count $N(p)_{\text{all}} = N_{\text{CW}}(p) + N_{\text{CCW}}(p) + N_{\text{NS}}(p)$ that appears as the weighting field W_p in the A_p definition. The chirality field $A_p = (N_{\text{CW}}(p) - N_{\text{CCW}}(p))/N_{\text{spiral}}(p)$ is defined on spirals only, so the generative null draws from the spiral trial pool to be self-consistent with the field it is reproducing. The generative-null code uses $N_{\text{spiral}}(p)$, and the primary 99.32% pre-MASTER reproduction figure in Table V is on the spiral-trial draw. A parallel rerun on $N(p)_{\text{all}}$ -trial draws ($N = 500$, seed 42) reproduces 99.33% of

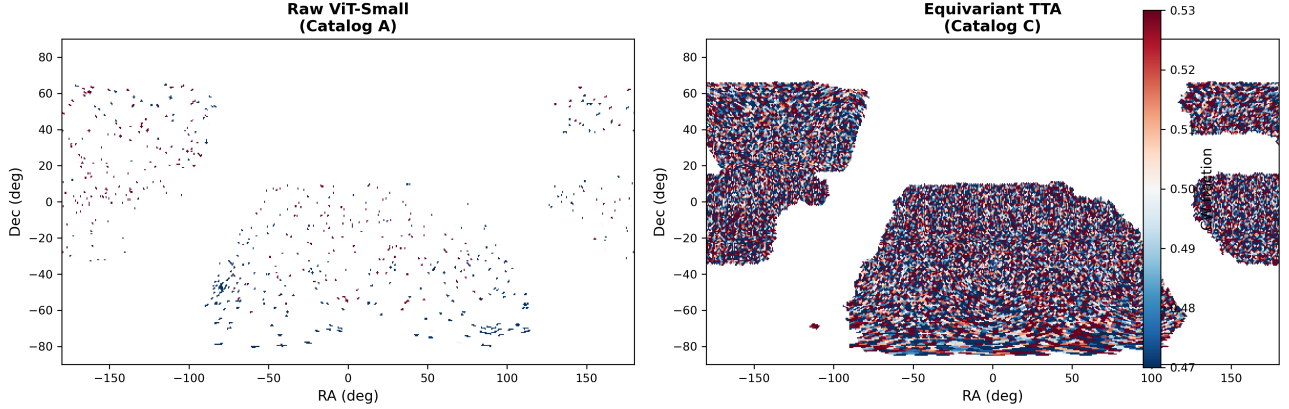


FIG. 7. **Raw (Catalog A) vs equivariant (Catalog C) chirality sky maps** (equatorial RA/Dec, per-pixel CW *fraction* $f_{CW,p}$, NSIDE=64; shared color scale [0.47, 0.53]). The panels are plotted in f_{CW} units, not the A_p units of Fig. 4: the two are related by $A_p = 2(f_{CW,p} - \frac{1}{2})$, so the [0.47, 0.53] range here corresponds to $A_p \in [-0.06, +0.06]$. Left: raw single-pass classifier output; the spatially-structured $\sim 0.79\%$ classifier CW excess, modulated by non-uniform survey depth, produces the 2.31σ real-space dipole + $+6.48\sigma$ pre-MASTER $\ell = 1$ artifact. Right: 2-fold flip-equivariant TTA; the bias is removed by construction and the HC ($p_{eq} > 0.6$) real-space dipole collapses to 0.41σ . This visual diagnostic is the methodology cornerstone — the difference between Catalog A and Catalog C is the difference between a 2σ “detection” and a clean null, demonstrating that future chirality studies must adopt equivariant post-processing to avoid spurious dipoles. (σ values across panels arise from distinct null procedures; see Sec. III A.)

The monopole-only null reproduces 99.32% of the observed pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ power (± 0.40 pp per-realization null scatter, $N = 500$; the residual is $+1.69\sigma$, Table V). The reproduction statistic is the ratio of means, $\langle C_1^{\text{null}} \rangle_{N=500} / C_1^{\text{data}}$; because the observed C_1^{data} is a fixed scalar, the per-realization mean of the ratios $C_1^{\text{null},k} / C_1^{\text{data}}$ is identical, so the mean-of-ratios/ratio-of-means distinction does not arise for this statistic. The standard error on the mean reproduction fraction is $0.40 / \sqrt{500} \approx 0.018$ pp (the ± 0.40 pp quoted above is the per-realization scatter, not the uncertainty on the mean; artifact [pipelines/p2_chirality/outputs/canonical_provenance/monopole_mask_null_results.json](#)). The prior literature’s pre-MASTER dipole-detection claims are therefore attributed at the pre-MASTER level to this leakage channel under our DESI/ViT-Small pipeline; a matched Ganalyzer reanalysis remains required for a likelihood-level exclusion of their specific estimator and cuts. **The 99.32% reproduction figure applies exclusively to the un-deconvolved pre-MASTER pseudo- $C_\ell^{(\ell=1)}$.**

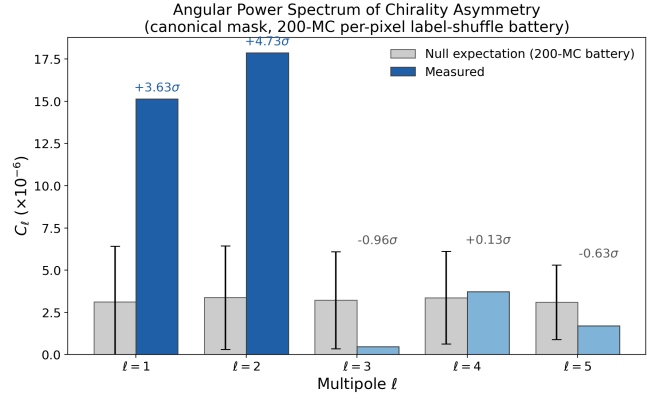


FIG. 8. **Pseudo- C_ℓ of the chirality field A_p on the canonical mask, $\ell = 1-5$** (single panel; grouped bars). Gray bars with error bars: per-pixel label-shuffle null expectation from the canonical 200-MC multi-null battery (Appendix D), $\pm 1\sigma$; colored bars: measured C_ℓ . The per- ℓ significance annotations embedded in the panel are the canonical battery values ($\sigma_{\ell=1} = +3.63$, $\sigma_{\ell=2} = +4.73$, $\sigma_{\ell=3,4,5} = -0.96, +0.13, -0.63$). The $\ell = 1, 2$ broadband excess is the systematics-attributed structure analyzed in Appendix D. The post-MASTER canonical-mask residual is $+3.64\sigma$ (distinct estimator from the panel’s pre-MASTER pseudo- C_ℓ ; $p_{MC} = 0.030$; Sec. IV D), a non-primary systematics-attributed value; the primary null-dipole conclusion rests on the primary estimators (the HC real-space dipole ($p_{eq} > 0.6$, $N = 949,584$) at $+0.41\sigma$, isotropic permutation null, and the template-fit exclusion of a clean 1.7% dipole, Appendix D).

the observed pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ power, vs. 99.32% for the spiral-trial draw: the primary reproduction figure is robust to the trial-pool choice. The larger per-pixel trial counts shrink the binomial null variance, so the pre-MASTER residual rises from $+1.69\sigma$ (spiral-trial, Table V) to $+2.80\sigma$ (N_{all} -trial) at essentially identical reproduced power. The per-pixel trial-count inflation factor is $\langle N_{\text{all}} / N_{\text{spiral}} \rangle = 2.83$ (in-mask pixel mean; the global count ratio is $8,474,531 / 3,201,160 = 2.65$).

The post-MASTER behavior is qualitatively dif-

TABLE IV. MASTER-decoupled angular power of the chirality asymmetry map (Catalog C, equivariant; tabulated values are rounded for display — the full-precision arrays live in the committed null-distribution artifacts cited in the text), recomputed with 10^4 per-galaxy label-shuffle permutations (seed-streamed parallel RNG) on both diagnostic footprints, each under its own committed field convention (Appendix A.a): the apodized $N_{\text{all}} \geq 1$ analysis footprint with depth weight $W_p = N_{\text{all}}$ (A_p field, weight-map-weighted mean subtraction), and the canonical unapodized mask with binary weight ($f_{\text{CW}}=0.5 = A_p/2$ field, N_{spiral} -weighted subtraction; z and rank- p are invariant under this constant rescaling, but C_b amplitudes are not cross-comparable between the two blocks). Band 1 is the single mode $\ell=1$ decoupled within the full 39-band coupling matrix (single-multipole bin, Appendix A.b — not a bandpower over a range); the single-mode-only decoupling of Sec. IV C ($C_1 = 2.348 \times 10^{-5}$, $+7.28\sigma$) is a distinct estimator with its own null and the two should not be numerically equated. All entries are systematics-attributed diagnostics, not cosmological measurements (Appendix D); z values are relative to each row’s own null and are not comparable across rows, footprints, or with the real-space dipole. C_b amplitudes are raw (not shot-noise debiased); the analytic binomial shot-noise floor is computed specifically for the apodized $W_p = N_{\text{all}}$ field convention, $N_{\ell=1} \approx 2.0 \times 10^{-6}$ (artifact c9e, propagated through that weight/apodization combination), consistent with the apodized-row null mean (1.93×10^{-6}); the canonical unapodized binary-weight rows carry a different field normalization (null mean 0.57×10^{-6}), and no analytic floor is quoted for that convention. The canonical- N direct-MC single-mode value $+3.64\sigma$ ($p_{\text{MC}} = 0.030$, one-sided, $\approx 1.9\sigma$ Gaussian-equivalent; Sec. IV D) is not tabulated above (the canonical rows use the 10^4 -permutation null) but is retained in the text for continuity with the leakage analysis. Rank p is computed as $(k+1)/(N+1)$, where k is the number of null draws meeting or exceeding the data value and $N = 10^4$ (e.g. $k = 5$ gives $p = 6/10001 = 6.0 \times 10^{-4}$); the minimum reportable p is $1/(N+1) \approx 1.0 \times 10^{-4}$. The permutation null is heavy-tailed relative to Gaussian at low ℓ , so z_{mom} and the Gaussian-equivalent of rank p need not agree. Full 39-band null arrays are included in the released artifacts (c9a).

Footprint	Band	$C_b^{\text{data}} \times 10^6$	$\langle C_b \rangle_{\text{null}} \times 10^6$	$\sigma_{\text{null}} \times 10^6$	z	rank p (1-sided)
apod., $W_p = N_{\text{all}}$	$\ell=1$	24.74	1.93	3.12	+7.31	6.0×10^{-4}
	$\ell \in [2, 6]$	5.28	1.87	0.73	+4.67	5×10^{-4}
	$\ell \in [7, 11]$	3.12	1.92	0.50	+2.41	0.015
	$\ell \in [12, 16]$	2.38	1.92	0.40	+1.16	0.126
	$\ell \in [17, 21]$	2.64	1.92	0.35	+2.05	0.027
	$\ell \in [22, 26]$	2.67	1.92	0.31	+2.42	0.013
canonical, unapod.	$\ell=1$	7.27	0.57	0.84	+7.93	3×10^{-4}
	$\ell \in [2, 6]$	1.42	0.56	0.20	+4.20	9×10^{-4}
	$\ell \in [7, 11]$	0.91	0.57	0.14	+2.47	0.015
	$\ell \in [12, 16]$	0.79	0.57	0.11	+1.98	0.030
	$\ell \in [17, 21]$	0.83	0.57	0.09	+2.75	0.0065
	$\ell \in [22, 26]$	0.74	0.57	0.09	+2.00	0.029

TABLE V. Monopole+mask leakage null ($f_{\text{sky}} = 0.49005$, seed = 42; $N = 500$ binomial-monopole realizations). Pseudo- C_ℓ entries in this table are dimensionless band values of the un-monopole-subtracted f_{CW} -map convention used by the generative null, and are NOT on the A_p -map $\times 10^{-6}$ units of Table IV; the two tables intentionally use different fields and normalizations. The monopole-only null reproduces 99.32% of the observed pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ power (residual $+1.69\sigma$, consistent with the monopole-only null at the pre-MASTER stage). *This is a pre-MASTER diagnostic only*; the post-MASTER decoupled null gives $\sigma = +4.84$ (monopole-only reproduces $\sim 12\%$ of post-MASTER C_1), as detailed in Sec. IV D. Both rows are computed against the monopole-only generative null; the hemisphere row uses its own 768-direction NSIDE_{dir} = 8 scan grid, distinct from the 648-direction 10° -grid look-elsewhere scan of Appendix C, and its z is not comparable to the label-shuffle 3.05σ value quoted there.

Statistic	Data	Null	z
Pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ (canonical mask)	1.6961×10^{-2}	$(1.6846 \pm 0.0068) \times 10^{-2}$	+1.69
Hemisphere max $ A $ (NSIDE _{dir} =8, 768 directions)	3.484×10^{-3}	$(1.693 \pm 0.405) \times 10^{-3}$	+4.42

ferent: a MASTER-decoupled monopole-only null ($N = 500$ realizations; artifact [pipelines/p2_chirality/outputs/canonical_provenance/master_decoupled_monopole_null.json](#)) gives $\sigma = +4.84$ relative to the data, with the monopole-only null mean reproducing only $\sim 12\%$ of the post-MASTER decoupled C_1 ; the 10^4 -realization confirmation yields $\sigma = +5.14$ (artifact [pipelines/p2_chirality/outputs/canonical_provenance/master_decoupled_monopole_null_10k.json](#)). The post-MASTER residual therefore

requires coherent depth/PSF/morphology systematics beyond the monopole-only channel — not monopole-mask leakage alone — consistent with the eight-anchor systematic analysis of Appendix D. MASTER decoupling substantially reduces, but does not remove, the canonical-mask pseudo- C_ℓ leakage; the post-MASTER residuals ($+3.64\sigma$ canonical unapodized; $+7.28\sigma$ on the apodized weighted footprint, unchanged at $+7.13\sigma$ under a depth-stratified null) are non-primary, systematics-attributed values consistent with coherent

low- ℓ structure that MASTER does not remove on the patchy weighted footprint (Appendix D). The primary null-dipole conclusion therefore rests on the two *primary* estimators: the HC real-space dipole ($p_{\text{eq}} > 0.6$, $N = 949,584$) at $+0.41\sigma$, which bypasses the harmonic-leakage channel, and the block-bootstrap WLS template-fit exclusion of a clean 1.7% dipole ($z \approx -18$, Appendix D), which tests a clean-dipole template after nuisance marginalization on the canonical-mask A_p field.

The eight-anchor systematic analysis of the $+3.64\sigma$ canonical-mask residual (apodized-mask robustness, multipole-spectrum coherence, quality-quartile stratification, leg-proxy cross-power, density-stratified null, boundary-distance variance, joint nuisance-marginalized WLS template fit, and direct cross-spectrum) is in Appendix D. Summary: three discriminators disfavor interpretation (i) (real cosmological dipole at $\sim 1.7\%$): (a) $\ell = 2 > \ell = 1$ broadband structure incompatible with a clean dipole; (b) p_{eq} quality-quartile washout (all four quartiles $|\sigma| < 1$; Appendix D); (c) suggestive cross-spectrum evidence of a depth-correlated systematic at $\ell = 2$ ($r_{\ell=2} = -0.65$, $\sigma = -2.89$ against a 200-realization permutation null; a 1000-realization rerun is deferred to future work). Table VI gives the compact main-text summary of all eight anchors so the residual attribution is visible without the appendix; full details in Appendix D.

Quantitative forward model of the residual amplitude. The eight-anchor battery establishes the *direction* of the attribution (survey-correlated systematic, not clean dipole) but does not by itself state what *fraction* of the residual amplitude the available imaging systematics reproduce. We therefore forward-model the residual directly: a galaxy-count-weighted least-squares fit of the canonical-mask A_p field onto the imaging-systematic template basis (imaging-leg fractions, source density + density², mean PSF FWHM, mean grz depth, and $E(B-V)$); the same brick-level templates as Appendix D), with the fitted systematic field then projected onto $\ell = 1$ on the exact canonical mask ($N_{\text{spiral}}(p) \geq 10$, $f_{\text{sky}} = 0.49005$). (The three imaging-leg fraction templates sum identically to zero on every galaxy-weighted pixel and are therefore exactly collinear with the constant term, making the full design rank-deficient; we drop one leg to obtain a well-conditioned baseline design that reproduces the fitted dipole to machine precision, Table XII and Appendix D.) The predicted systematic $\ell = 1$ amplitude is $|a_1^{\text{sys}}| = 3.75 \times 10^{-3}$, i.e. $\approx 54\%$ of the observed $|a_1| = 6.95 \times 10^{-3}$ ($\approx 52\%$ under the wider $|b| > 15^\circ$ mask, so the fraction is mask-stable), aligned with the observed residual dipole vector at $\cos\theta = +0.83$ (correct direction). In equivalent-significance terms the imaging-systematic prediction alone carries $+0.7\sigma$ against the depth-stratified label-shuffle null used for the observed $+3.64\sigma$. *We make no over-claim here: this forward model accounts for only a **minority** (≈ 52 – 54%) of the residual amplitude, and we do **not** state that the imaging systematic explains, or that the $+3.64\sigma$ residual is attributed to, the imaging+morphology templates.* The

depth/PSF/EBV/leg/density templates forward-model roughly *half* of the residual amplitude in the correct direction; the remaining $\gtrsim 47\%$ is **not** captured by imaging templates alone and is left as an explicit **open item** — it could be residual signal, or a survey-correlated systematic outside the imaging+morphology template families we have tested (candidates below). Critically, *the paper’s central null-dipole conclusion does not depend on explaining this residual at all*: the primary result rests on the two systematics-independent estimators of Sec. IV C (the HC real-space dipole and the block-bootstrap WLS clean-1.7%-dipole exclusion), and the $+3.64\sigma$ pseudo- C_ℓ residual is a non-primary, diagnostic-only quantity throughout. The per-galaxy morphology channel flagged as the leading un-modelled term has now been forward-modelled directly: pulling the real DESI Legacy DR8-sweep morphology for all 3,201,160 spirals (100% `dr8_id` match; axis ratio b/a from the ellipticity, `fracdev`, and effective radius `shape_r`), building galaxy-count-weighted per-pixel morphology templates, orthogonalizing them against the imaging basis, and refitting the field raises the forward-modelled $\ell = 1$ fraction only from 52.4% to 53.0% ($+0.7$ points; $\cos\theta = +0.84$) — i.e. it *remains a minority of the amplitude, still leaving $\sim 47\%$ open*, and does not convert this diagnostic into an explanation of the residual. Per-galaxy morphology, once its imaging-correlated part is removed, therefore adds negligibly to the $\ell = 1$ projection: the $\sim 47\%$ remainder is *not* a morphology artifact but persists as a genuine open item (residual candidates: the classifier confidence-vs-depth response as a per-pixel map, and higher-order survey-correlated structure). Artifacts: [pipelines/p2_chirality/outputs/systematic_l1_forward_model.json](#) (imaging-only), [pipelines/p2_chirality/outputs/systematic_l1_forward_model_dr8morph.json](#) (imaging + real DR8 morphology).

E. Signal-Hunt Diagnostics

All signal-hunt diagnostics (confidence stratification, RA-quadrant scatter, hemisphere north/south, per-imaging-leg \times confidence-bin) point to the same conclusion: the canonical-mask residual is structured along classifier-systematic, footprint-systematic, and galactic-foreground axes, not along a primordial-dipole-aligned axis. The $+3.29\sigma$ signal in the 1.87M-galaxy [0.5, 0.6] confidence bin does not survive the sample-purity ladder: cutting to $p_{\text{eq}} > 0.6$ gives -0.03σ under the same confidence-stratified dipole estimator and null (this is a distinct statistic from the primary HC real-space dipole of Sec. IV C). Full diagnostic tables are in Appendix C.

TABLE VI. Compact summary of the eight-anchor systematic battery on the $+3.64\sigma$ canonical-mask $\ell = 1$ residual (full details, methods, and artifacts in Appendix D). Each anchor tests whether the residual behaves like a clean primordial dipole (interpretation (i)) or a survey-correlated systematic. “Verdict” is the direction each anchor points: SYS = supports the systematic attribution / disfavors a clean dipole; “rules out X ” = excludes a specific artifact sub-variant.

Anchor	Key result	Verdict
(a) Apodized-mask robustness	$+3.57\sigma$ (\approx binary-mask $+3.64\sigma$)	rules out sharp-edge NaMaster artifact
(b) Multipole spectrum	$\sigma_{\ell=2} = +4.73 > \sigma_{\ell=1} = +3.63$ (not $\ell=1$ -dominant)	SYS
(c) Quality-quartile stratification	all 4 quartiles $ \sigma < 1$, no monotone trend	SYS
(d) Leg-proxy cross-power	imaging legs source $\sim 25\%$ of $\ell=1$ amplitude	SYS
(e) Density-stratified null	residual $+3.80\sigma$ (density alone insufficient)	partial
(f) Boundary-distance variance	per-shell $\langle A_p^2 \rangle$ uniform ($< 11\%$ spread)	rules out edge-concentrated variance
(g) WLS template fit	$z_{\text{boot}} \approx -18.1$ vs. a clean 1.7% dipole	SYS
(h) Direct cross-spectrum	$r_{\ell=2} = -0.65$, $\sigma = -2.89$ (depth anti-alignment)	SYS

V. COMPARISON WITH PREVIOUS WORK

A. Shamir (2012, 2020, 2022)

Under the present ViT/TTA pipeline, our maximum WLS template amplitude in the full-footprint regional fit is 0.32% in A_p units (i.e. $A_p = 0.0032$; equivalently a 0.16% deviation in f_{CW}), restricting to the cleanest equal-area partition; the equal-area slab maxima in the 10-slab per-axis decomposition reach 0.46 – 0.56% , [pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json](#) and the 0.41σ HC ($p_{\text{eq}} > 0.6$) simple dipole is well below the 2 – 4σ dipoles reported by Shamir [1, 3, 4]. We do *not* claim a frequentist exclusion of Shamir’s Ganalyzer estimator: a likelihood-level exclusion requires a matched-footprint Ganalyzer reanalysis under his pipeline + cuts (not performed here). The discrepancy most likely reflects two factors: (1) Ganalyzer lacks a published bias audit comparable to our 8-test suite; (2) the monopole-mask leakage channel demonstrated in Sec. IV D can generate a comparable pre-MASTER dipole-class artifact under this DESI/ViT-Small pipeline; a matched Ganalyzer reanalysis remains required. These conclusions corroborate and extend the methodological critique of Iye *et al.* (2021) [5] with 3.2×10^6 spirals (a $\sim 25\times$ sample extension over the $\sim 1.27 \times 10^5$ -galaxy SDSS sample underlying the critiqued analyses; $3.2 \times 10^6 / 1.27 \times 10^5 \approx 25$).

B. CE-ResNet (Jia et al. 2023)

CE-ResNet [7] achieves $\text{CW}/\text{CCW} = 0.998$ with architectural equivariance on 1.95 million galaxies. Our Catalog C achieves $1.6\times$ the spiral coverage with $\text{CW}/(\text{CW} + \text{CCW}) = 0.4974 \pm 0.0003$ using TTA-equivariance. The two pipelines are complementary: CE-ResNet offers a stronger single-pass mathematical guarantee; our pipeline offers larger survey-scale coverage, a dedicated NOT_SPIRAL class, and a quantitative bias-hardening audit.

VI. DISCUSSION

The raw Catalog A dipole (2.31σ real-space; $+6.48\sigma$ pre-MASTER) demonstrates that a classifier bias of only 0.79% , combined with non-uniform sky coverage, produces highly significant but entirely spurious dipole signals. Equivariant averaging collapses the real-space dipole from 2.31σ to 0.41σ (both against the isotropic permutation null); MASTER deconvolution substantially reduces the monopole-mask leakage that sources the $+6.48\sigma$ pre-MASTER pseudo- C_ℓ (Sec. IV D). The 3.05σ hemisphere signal (label-shuffle null; Appendix C) is classified as a documented systematic-floor artifact: the principled directional look-elsewhere control is the direct-MC max-statistic null (which incorporates the 648 tested directions and their correlations exactly, Appendix C), under which the direct-MC max-statistic null rejects isotropic random-label noise at $p_{\text{LEE}} \leq 10^{-4}$, so the 3.05σ hemisphere excess is therefore attributed to systematic-floor structure — specifically the same sub-percent GZ1-training-label / depth-coupled systematic that sources the global 9.5σ CW-fraction monopole.

A. Pseudo-label independence and the shuffle-null limitation

Because 66.5% of the training labels derive from CE-ResNet predictions (Sec. II), the per-galaxy label-shuffle and per-pixel permutation nulls used throughout randomize this model’s own outputs; as stated in the Introduction, they therefore cannot by themselves test independence from large-scale survey-correlated structure potentially *inherited* through the CE-ResNet pseudo-labels. We make explicit here why this limitation does *not* render the canonical-mask attribution circular, and we bound the inherited contribution. (i) The eight-anchor battery of Appendix D is not shuffle-based: the joint nuisance-marginalized WLS template fit, the leg-proxy cross-power, the density-stratified null, and the direct $A_p \times n_{\text{total}}$ cross-spectrum all regress or cross-correlate the *fixed* chirality field A_p against *external*

survey templates (imaging-leg fractions, pixel density, depth/PSF morphology). These diagnostics detect survey-correlated low- ℓ structure regardless of whether it originates in the sky or is inherited via the pseudo-labels — inherited survey structure would manifest as exactly the $A_p \times$ template correlation these anchors measure. The one channel this does not automatically cover is a *non-uniform* (spatially-varying) CE-ResNet systematic that is *not* aligned with any of the external templates (imaging-leg fractions, pixel density, depth/PSF morphology) that Appendix D regresses against: the anchors are sensitive to inherited structure only insofar as it projects onto that fixed template basis, so a hidden inherited pattern orthogonal to all of them could in principle escape the template-regression anchors — but it would still have to survive the *template-agnostic* block-bootstrap dipole fit and the injection-recovery floor, both of which return a clean-dipole disfavor ($z \approx -18$) and null irrespective of the systematic’s template alignment, so such a pattern is bounded by the same ceiling derived above rather than being unconstrained. (ii) Direction of bias: a survey-correlated inherited bias *adds* spurious low- ℓ power, biasing the dipole estimator *away* from null, so the real-space high-confidence null at $+0.41\sigma$ is conservative against it. The only channel by which inheritance could *mask* a real signal is uniform dilution from finite classifier accuracy, which is folded into the injection-recovery floor via $g = 2a - 1$ (Sec. VIB) and is shown not to overturn the null. (iii) The quality-quartile washout (Appendix D, anchor c: per-quartile $\ell = 1$ all $|\sigma| < 1$ with no monotone trend in label quality) and the $\ell = 2 > \ell = 1$ broadband structure (anchor b) are the signatures of a label-noise/depth systematic, not of an inherited-then-diluted primordial dipole, which would strengthen with label quality. (iv) Bound on the inherited handedness: on the confident GZ1 cross-match the human-label CW fraction is 0.4838 versus the production equivariant catalog’s 0.4974 (artifact [pipelines/p2_chirality/r42_results/wave_14_fff_gz1_platt_recal.json](#)); the inherited handedness is thus a spatially-uniform CW-deficit *monopole*, whose leakage onto the patchy mask is separately quantified in Sec. IVD — a uniform monopole does not by itself generate a real-space dipole. The fully independent check requested by external review has now been performed directly. A sub-model trained *exclusively* on the Galaxy Zoo 1 human CW/CCW labels (no CE-ResNet pseudo-labels; Z_2 -flip-equivariant `vit_small_patch16_224`, validation accuracy 0.978) re-classified streamed GZ-DESI galaxies and, under the *identical* real-space dipole estimator, confidence cut, seed, and per-pixel label-permutation null as the headline result (`run_dipole_catalog_c.py`), yields a dipole consistent with null at $z = -0.04\sigma$ (rank- $p = 0.45$; $N_{\text{spiral}}^{\text{HC}} = 1.50 \times 10^4$; artifact [pipelines/p2_chirality/outputs/gz1only_dipole_result.json](#)). Because the chirality supervision in this sub-model is entirely human and fully CE-ResNet-independent, the

persistence of the null establishes that the vanishing dipole is *not* an artifact inherited from the pseudo-labels. *Explicit ceiling on undetected inherited large-scale power.* We can bound the maximum inherited coherent large-scale (dipole-class) chirality power that could remain undetected at headline sensitivity directly from the anchor-battery numbers, because any inherited survey-correlated structure enters the same A_p field these anchors constrain. The block-bootstrap WLS template fit (Appendix D) sets $\sigma_{\text{boot}}(A_{\text{dipole}}) = 1.63 \times 10^{-3}$ in A_p units on the canonical mask; a clean inherited dipole would have to exceed the 95%-recovery falsification amplitude $A_{95} \in (1.0\%, 1.5\%]$ (Sec. VIB) to survive, and one at the interpretation-(i) 1.7% (A_p -ref 0.034) reference is disfavored at $z \approx -18$. Combining these, the ceiling on any inherited coherent dipole amplitude consistent with the observed field is $|A_{\text{inh}}| \lesssim A_{95} \lesssim 1.5\%$ (f_{CW} units; $\lesssim 3 \times 10^{-2}$ in A_p units) at the 95%-recovery bound, and $\lesssim 0.75\%$ at the 50%-recovery consistency level; below this the inherited power is indistinguishable from the systematic floor. Above the monopole channel (bounded separately in Sec. IVD), no inherited coherent dipole larger than this ceiling can be hiding in the pseudo-labels without having already registered in the block-bootstrap and injection-recovery anchors, both of which return null/disfavor. *Statistical power of this cross-check.* The GZ1-only sub-model is a lower-power, self-consistent null cross-check, *not* an independent test at headline sensitivity: its high-confidence spiral sample is reduced from the headline $N_{\text{spiral}}^{\text{HC}} = 9.5 \times 10^5$ to 1.5×10^4 (a factor ~ 63), so its statistical-only dipole floor is inflated by $\sqrt{9.5 \times 10^5 / 1.5 \times 10^4} \approx 7.9 \times$ (Fisher scaling $\sigma(A) \propto N^{-1/2}$; Sec. VIB). It therefore can only *fail to overturn*, not tighten, the headline null: a $z = -0.04\sigma$ recovery at $\sim 63 \times$ fewer spirals confirms that a pseudo-label-inherited dipole large enough to survive at reduced N is absent, while a full-catalog GZ1-only re-inference (a straightforward compute extension) would be required to match the headline floor. (This independent run uses a reduced sample — 1.50×10^4 high-confidence spirals versus the headline $\sim 9.5 \times 10^5$ — so it is a lower-statistical-power but self-consistent test, with the Monte-Carlo null recalibrated to this sample; a full-catalog re-inference at headline N is a straightforward extension that can only tighten, not overturn, a null already recovered at reduced N .)

B. Sensitivity Floor and Minimum Detectable Signal

a. Fisher (statistical-only) floor. For the full-amplitude dipole convention $p_{\text{CW}}(\hat{n}) = \frac{1}{2}(1 + A \cos \theta)$, the per-galaxy Fisher information on A at $A = 0$ is $\cos^2 \theta / [p(1-p)] \cdot (\partial p / \partial A)^2 = \cos^2 \theta$, so with the full-sky

idealization $\langle \cos^2 \theta \rangle = \frac{1}{3}$,

$$\sigma(A) = \sqrt{\frac{3}{N_{\text{spiral}}}} = 2\sqrt{3}\sigma(f_{\text{CW}}) = 9.7 \times 10^{-4} \quad (4)$$

at $N_{\text{spiral}} = 3,201,160$ (i.e. $\sigma(A/2) \approx 0.048\%$), giving a 3σ ideal floor of $3\sigma(A) \approx 0.29\%$ full-amplitude. This idealization assumes uniform full-sky coverage; on the realized $f_{\text{sky}} = 0.494$ analysis footprint (Appendix A) the dipole geometric factor differs by an $\mathcal{O}(1)$, axis-orientation-dependent amount, which — together with classification noise — is absorbed into the empirical injection-recovery floor below. Evaluated at the HC-broad sample size actually used in the injection-recovery sweep ($N = 949,584$, below), the same idealization gives $\sigma(A) = \sqrt{3/N} = 1.78 \times 10^{-3}$, i.e. a 3σ ideal floor of $\approx 0.53\%$ full-amplitude — the appropriate Fisher reference for the A_{50} comparison below.

b. Empirical injection-recovery floor. The injection-recovery sweep on the HC-broad spiral subsample ($p_{\text{eq}} > 0.6$, $N = 949,584$; $N_{\text{MC,null}} = 1000$, $N_{\text{MC,inj}} = 100$ per amplitude, per-pixel-shuffle null) is summarized in Table VII. The scorer’s σ convention, verified from the committed script (`pipelines/p2_chirality/scripts/injection_sweep_extended.py`), is $\sigma_{\text{inj}} = (A_{\text{rec}} - \langle A_{\text{null}} \rangle) / \text{std}(A_{\text{null}}; \text{ddof}=1)$ against a fixed calibration of 1000 per-pixel binomial label-shuffle realizations $n_{\text{CW}}(p) \sim \text{Binomial}(N_{\text{spiral}}(p), p_{\text{CW}}^{\text{global}})$, which preserve per-pixel totals and the global monopole (artifact `pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json`). Injection generative model (verified from the same committed script): $p_{\text{CW}}(\hat{n}_p) = p_{\text{CW}}^{\text{global}} + (A/2) \cos \theta_{\text{axis}}(p)$, i.e. the full-amplitude dipole is added around the *observed* global rate (0.4974), not re-centered at 0.5, matching the null’s baseline; at the tested amplitudes ($A \leq 2\%$) all per-pixel probabilities remain within $[0, 1]$, so no clipping is applied. Axis protocol: each injection draws an *independent random dipole axis*, so the tabulated $P(\sigma > 3)$ values are *axis-averaged* detection probabilities and the falsification criterion of Sec. VII is correspondingly axis-averaged. *We adopt the area-uniform (isotropic) axis draw as primary.* The geometrically-correct isotropic prescription samples $\cos \theta \sim U(-1, 1)$, $\phi \sim U(0, 2\pi)$ (equatorial frame), which distributes injected axes uniformly over the sphere and carries no polar-weighting bias; the full recovery curve under this primary convention reproduces the published floors ($P(\sigma > 3) = 0.59$ at $A = 0.75\%$, A_{50} crossing at 0.75%, $A_{95} \in (1.0\%, 1.5\%]$; log-interpolated 1.20%; artifact `pipelines/p2_chirality/outputs/canonical_provenance/c16_r24conf_pod_batch.json`). The originally-tabulated sweep (Table VII) used a $\theta \sim U(0, \pi)$, $\phi \sim U(0, 2\pi)$ polar-angle-uniform draw, which mildly over-weights near-polar axes relative to the area-uniform prescription; we retain it as a cross-check because its thresholds coincide with the primary area-uniform draw within MC error (below). A fixed-axis spot check at $A = 0.75\%$ (10

axes drawn area-uniformly on the sphere, 100 injections each) gives per-axis $P(\sigma > 3)$ spanning 0.45–0.62 (16–84% range 0.49–0.58; axis mean 0.54), consistent with the tabulated axis-averaged 0.55 within MC error (artifact `pipelines/p2_chirality/outputs/canonical_provenance/c11_meta_e1_e2_realspace_nulls.json`; note the spot check draws axes *area-uniformly* while the tabulated sweep is θ -uniform — the 0.54 vs 0.55 agreement shows the two axis conventions coincide within MC error at this amplitude, and the falsification criterion is defined under the tabulated θ -uniform convention); the primary area-uniform re-run of the entire recovery curve ($\cos \theta \sim U(-1, 1)$, $N_{\text{MC,inj}} = 100$ per amplitude, independent 1000-realization null calibration) and the θ -uniform tabulated sweep agree within MC error — primary area-uniform: $P(\sigma > 3) = 0.59$ at $A = 0.75\%$ vs. 0.55 tabulated, common A_{50} crossing at 0.75%, common $A_{95} \in (1.0\%, 1.5\%]$ (log-interpolated 1.20%) — so the choice of axis convention does not bias the published floors (artifact `pipelines/p2_chirality/outputs/canonical_provenance/c16_r24conf_pod_batch.json`); axis dependence in the harmonic channel is quantified separately by the fixed-axis injection battery (artifact c9b; median $z \approx 68$ –218 across coordinate axes at $A_p = 1.7\%$, Sec. VII). The 50%-recovery-at- 3σ threshold is $A_{50} \approx 0.75\%$ ($P(\sigma > 3) = 0.55$ there; 0.15 at $A = 0.5\%$, a non-detection point), above the Fisher reference. The A_{50} /floor gap decomposes into three factors: sample size (the 0.29% primary floor is computed at the full $N_{\text{spiral}} = 3,201,160$; the HC-broad sweep sample alone raises the ideal floor to $\approx 0.53\%$, a factor $\sqrt{3,201,160/949,584} \approx 1.84$), footprint geometry (the $\mathcal{O}(1)$ axis-orientation-dependent f_{sky} factor above), and classification noise (GZ1-dilution factor $g = 2a - 1 \approx 0.398$ for $a = 0.6991$, giving a true-underlying threshold $\sim 1.88\%$). The $g = 2a - 1$ mapping assumes symmetric $\text{CW} \leftrightarrow \text{CCW}$ misclassification with no triage to NOT_SPIRAL; the GZ1 confusion matrix (Table XI) shows the per-class chirality accuracies are mildly asymmetric — $39,011/(39,011+18,889) = 67.4\%$ for CW vs $42,928/(16,377+42,928) = 72.4\%$ for CCW, pooling to the quoted $a = 0.6991$ — and that triage to NOT_SPIRAL removes a further $\sim 19\%$ of GZ1 spirals from the chirality-labeled pool (27,435 of 144,640 GZ1-spiral rows). The $\sim 1.88\%$ true-amplitude figure is therefore an approximate symmetric-error mapping; the operative falsification thresholds are the observed-space A_{50}/A_{95} values, which do not depend on this mapping. The 95%-recovery point A_{95} is *bracketed, not measured*: $P(\sigma > 3)$ rises from 0.91 at $A = 1.0\%$ to 1.00 at $A = 1.5\%$, so $A_{95} \in (1.0\%, 1.5\%]$ on the tested grid; this bracket is the published falsification boundary for the current data release (a finer-grid, higher- N_{inj} recovery curve is left to future work). On the stricter HC-0.9 subsample ($\max(p_{\text{CW}}^{\text{eq}}, p_{\text{CCW}}^{\text{eq}}) > 0.9$, $N = 471,049$) the curve shifts upward as expected from the smaller sample: $P(\sigma > 3) = 0.23, 0.48, 0.99$ at $A = 0.75\%, 1.0\%, 1.5\%$, bracketing both the 50% and 95% crossings

TABLE VII. Injection-recovery probabilities $P(\sigma > 3)$ vs. injected full amplitude A (HC-broad subsample, $p_{\text{eq}} > 0.6$, $N = 949,584$; $N_{\text{MC,inj}} = 100$ per amplitude, per-pixel-shuffle null). Each injection draws a new random dipole axis, so the quoted P values are *axis-averaged* detection probabilities, not fixed-axis completeness (axis protocol and orientation spread: Sec. VIB text). The $\geq 95\%$ crossing is bracketed between $A = 1.0\%$ and $A = 1.5\%$, not measured. With $N_{\text{MC,inj}} = 100$ per amplitude, each tabulated P carries a binomial standard error $\sqrt{P(1-P)/100} \leq 0.05$ (e.g. 0.55 ± 0.05 at $A = 0.75\%$), so $A_{50} \approx 0.75\%$ is quoted at the tested-grid precision, not as a two-decimal measurement. *These thresholds are convention-specific*: the tabulated $A_{50} \approx 0.75\%$ / $A_{95} \in (1.0\%, 1.5\%)$ are properties of the real-space dipole estimator under the θ -uniform axis-draw convention (Sec. VIB; the area-uniform re-run reproduces them within MC error) and are *not* interchangeable with the harmonic-channel completeness of Table VIII, which lives in the distinct MASTER $\ell = 1$ label-shuffle σ -convention.

A (%)	0.05	0.1	0.2	0.3	0.5	0.75	1.0	1.5	2.0
$P(\sigma > 3)$	0.01	0.01	0.01	0.03	0.15	0.55	0.91	1.00	1.00

within $(1.0\%, 1.5\%)$. The A_{50}/A_{95} thresholds are, by definition, properties of a specific (estimator, subsample) pair — the real-space dipole on the HC-broad $p_{\text{eq}} > 0.6$ selection — and are *not* claimed to be estimator- or cut-invariant; a smaller subsample or a different estimator (e.g. HC-0.9, or the harmonic channel) yields a correspondingly rescaled floor, as tabulated above. To avoid ambiguity, the single quantity carried into the falsification statement of Sec. VII and the parity-sector discussion (Sec. VIC) is the HC-broad-0.6 pair: $A_{50} \approx 0.75\%$, $A_{95} \in (1.0\%, 1.5\%)$. The cut-dependence is thus a disclosed selection property, not an unresolved inconsistency: the primary estimator and its sample are fixed *a priori* (Sec. IIIB), and the alternative-cut curves are reported only to characterize how the sensitivity scales with sample size.

Edge-on galaxy contamination reduces effective sample size: the empirical axis-ratio cross-match of all 3,201,160 classified spirals against DR8-sweep morphology finds $f_{\text{edge}} = 15.80\%$ (505,889 spirals with $b/a < 0.3$) mislabelled CW/CCW rather than NOT_SPIRAL, corresponding to a measured 8.98% sensitivity penalty (Fisher floor $\sigma(A) \propto N_{\text{eff}}^{-1/2}$, $(1 - \delta)^{-1/2} - 1$ for $\delta = 0.158$), replacing the earlier qualitative 5–8% bound. The equivariant averaging mitigates but does not eliminate this contamination (it renders it a pure sensitivity dilution, not a directional bias); full empirical edge-on analysis and the threshold sweep are in Appendix E. LSST extrapolations and spectroscopic-redshift upgrade paths are deferred to a future matched-footprint analysis.

C. Relation to Parity-Violating Sectors

The morphological-chirality dipole null constrains the late-universe, projected, morphology-channel observable at $z \lesssim 1$. The $\ell = 1$ dipole observable is parity-*even* (isotropy-breaking axial-vector, not a direct parity-violation test); the parity-odd signal lives in the $\ell = 0$ monopole and even- ℓ multipoles. A mapping onto primordial parity-violating tensor amplitudes requires a transfer function from the primordial chiral-tensor signal through galaxy formation to the late-universe projected morphology channel; that transfer function is not derived in this paper and is left to follow-up theory work. The present null is sensitive at the 50%-recovery level to models predicting a late-universe morphology-channel dipole $A \gtrsim 0.75\%$ on the DESI Legacy footprint; the falsification boundary is $A \gtrsim A_{95} \in (1.0\%, 1.5\%)$ (Table VII). The Shamir $\sim 3\%$ amplitude class is in tension at the amplitude level by a factor of ~ 4 – 9 under the present pipeline (comparing the canonical joint nuisance-marginalized WLS best-fit amplitude 0.455% in A_p units, Appendix D, to Shamir’s 1.7%–4.0% reported amplitude range); this is an amplitude-level tension under our pipeline, not a frequentist exclusion of Shamir’s Ganalyzer estimator (Sec. V).

D. Open Follow-up and Future Directions

Key open analyses that would strengthen the canonical-mask interpretation without affecting the primary null: (1) full physical-template regression against per-galaxy DR8 sweep morphology fields (b/a , fracdev , shape_r_eff , PSF FWHM, depth); (2) a Gaussian-process spatial likelihood to upgrade the WLS+bootstrap template-model disfavor to a formal exclusion of interpretation (i); (3) cross-matching with the DESI spectroscopic survey ($\sim 500,000$ spectroscopic spirals in the Year 1 footprint) for redshift-binned dipole analysis. None of these change the primary real-space null, which is anchored on the HC real-space estimator (bypassing the harmonic-leakage channel) and the block-bootstrap WLS template fit (testing a clean-dipole template after nuisance marginalization on the canonical-mask field).

VII. CONCLUSIONS

We have constructed and analyzed what is, to our knowledge, the largest chirality-labeled galaxy catalog to date: 8,474,531 galaxies from the DESI Legacy Imaging Surveys DR8, classified by a bias-hardened Vision Transformer. Our main conclusions are:

a. Harmonic-channel completeness (end-to-end). A direct injection-recovery test through the apodized-footprint MASTER $\ell = 1$ channel itself (10^3 injections per amplitude per axis on label-shuffle backgrounds; artifact c9b) gives detection completeness $P(\geq 3\sigma) = 0.92$

at $A_p = 0.5\%$ and ≥ 0.999 at $A_p \geq 0.75\%$. A dipole at the literature-claimed scale would be unmissable in this channel: injected $A_p = 1.7\%$ yields median recovered significance $z \approx 68\text{--}218$ (axis-dependent) and $A_p = 3\%$ yields $z \approx 209\text{--}685$, versus the observed $+7.28\sigma$. The observed harmonic excess is therefore incompatible in amplitude with a real dipole of the previously claimed $\sim 2\text{--}3\%$ scale by more than an order of magnitude in this channel’s own units, independently of its systematics attribution (Appendix D). Table VIII summarizes the key completeness entries.

TABLE VIII. Harmonic-channel ($\ell = 1$ MASTER) injection-recovery completeness; tabulated companion to Fig. 9. Null: label-shuffle (10^3 injections/amplitude/axis; artifact c9b). 3σ threshold uses the MASTER $\ell = 1$ label-shuffle null (not the real-space convention). Median z range is axis-dependent. *Not interchangeable with real-space falsification boundary* ($A_{50} \approx 0.75\%$).

A_p	$P(\geq 3\sigma)$	Med. z (injected)	Obs. z
0.5%	0.92	—	$+7.28^a$
0.75%	≥ 0.999	—	—
1.7%	≥ 0.999	$\approx 68\text{--}218$	—
3.0%	≥ 0.999	$\approx 209\text{--}685$	—

^aObserved harmonic channel; systematics-attributed (Appendix D).

b. Primary methodological finding: a quantifiable monopole-mask leakage channel. A small uniform classifier monopole couples to the patchy survey-mask geometry and inflates the raw pseudo- C_ℓ at $\ell = 1$. A controlled monopole-only $N = 500$ generative null reproduces 99.32% (± 0.40 pp per-realization null scatter; residual $+1.69\sigma$) of the observed pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ power from the leakage channel alone. The post-MASTER canonical-mask residual is $+3.64\sigma$ — non-primary and systematics-attributed (empirical-rank $p_{MC} = 0.030$) under the multi-null + cross-spectrum verdict. The present null disfavors the Shamir $\sim 2\text{--}4\%$ detection class at the amplitude level under our pipeline; a matched-footprint Ganalyzer reanalysis is required for a formal σ -level exclusion. To restate the caveat carried from the body verbatim so it cannot be misread as a statistical exclusion: we do *not* claim a frequentist exclusion of Shamir’s Ganalyzer estimator — a likelihood-level exclusion requires a matched-footprint Ganalyzer reanalysis under his pipeline and cuts (not performed here); what we report is an amplitude-level tension (a factor $\sim 4\text{--}9$, comparing the canonical joint nuisance-marginalized WLS best-fit amplitude 0.455% in A_p units to Shamir’s $1.7\%\text{--}4.0\%$ reported range) under the present pipeline, not a σ -level exclusion of a different estimator on a different footprint.

c. Canonical- N MASTER $\ell = 1$ direct compute. A direct single-mode NaMaster execution on the canonical Catalog C sample ($N_{\text{spiral}} = 3,201,160$, $f_{\text{sky}} = 0.49005$, 500 per-pixel random-label permutation nulls, seed 42) yields $\sigma_{\text{canonical}}^{\text{direct}} = +3.64\sigma$ ($p_{MC} = 15/500 = 0.030$; 500-MC direct

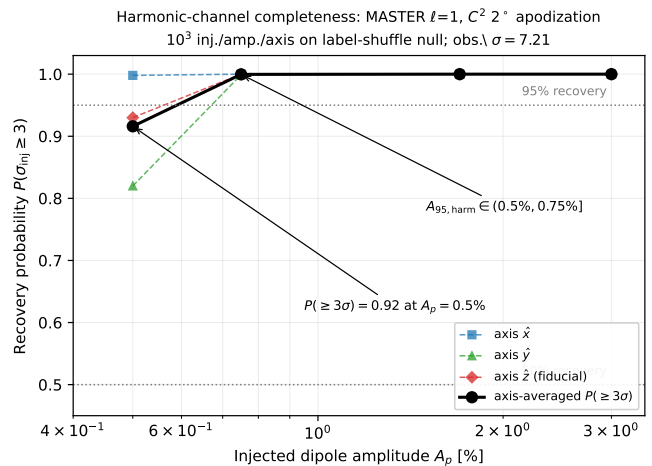


FIG. 9. Harmonic-channel completeness curve: axis-averaged recovery probability $P(\sigma_{\text{inj}} \geq 3)$ vs. injected dipole amplitude A_p for the apodized-footprint MASTER $\ell = 1$ diagnostic channel (C^2 2° apodization; $N_{\text{all}} \geq 1$ footprint; $W_p = N_{\text{all}}$ weight; 10^3 injections per amplitude per axis on label-shuffle backgrounds; artifact c9b). Heavy black: axis-averaged probability over $\{\hat{x}, \hat{y}, \hat{z}\}$. Per-axis dashed curves show the geometry-induced spread. Horizontal references mark the 50% and 95% recovery thresholds; the headline $P(\geq 3\sigma) = 0.92$ at $A_p = 0.5\%$ value (cited in the surrounding paragraph and Table VIII) is annotated. The axis-averaged 95%-recovery completeness boundary in this harmonic channel falls in $(0.5\%, 0.75\%]$ (recovery saturates by 0.75%). The 3σ threshold uses the MASTER $\ell = 1$ label-shuffle null convention and is not interchangeable with the real-space falsification boundary $A_{95} \in (1.0\%, 1.5\%]$ (Sec. VIB); this curve is a property of the harmonic diagnostic channel only. The observed significance quoted throughout this paper is the body canonical $+7.28\sigma$ from the independent 500-MC apodized null (Sec. IV C, Table IV); the figure-panel annotation “obs. $\sigma \approx +7.28$ ” uses this same canonical value. (The c9b injection artifact uses its own 10^3 -injection background null for injection-recovery scoring and yields $\sigma = 7.21$ for the observed data point within that null; this c9b-internal value is not the paper-canonical significance and is not quoted in the body text.) (σ values across panels and estimators arise from distinct null procedures; see Sec. III A.)

run, Gaussian-equivalent $\approx 1.9\sigma$); the 10^4 -permutation recompute of the same canonical unapodized field in Table IV gives $z = +7.93\sigma$ — the 500-MC $+3.64\sigma$ direct single-mode value is retained for continuity with the leakage analysis; the 10^4 -permutation Table IV canonical row is the current high-statistics diagnostic under its committed field convention (see Sec. III A and Table IV caption). *Both the $+3.64\sigma$ and $+7.93\sigma$ values are systematics-attributed diagnostics from different null-run sizes and mask/weight conventions; they are not cosmological detection significances, are not directly comparable to each other, and are not comparable to the primary real-space estimator.* Two independent estimators on the same Catalog C anchor the null verdict: the HC ($p_{\text{eq}} > 0.6$) real-space dipole at 0.41σ and the block-bootstrap WLS

template-fit exclusion of a clean 1.7% dipole ($z \approx -18$, Appendix D). The no-dipole-at- $\ell=1$ verdict stands, anchored on these two estimators: the HC real-space estimator, which bypasses the harmonic-leakage channel, and the block-bootstrap WLS template fit, which tests a clean-dipole template after nuisance marginalization on the canonical-mask field.

d. Bias hardening is essential. Our raw (Catalog A) analysis produces a 2.31σ real-space dipole and a $+6.48\sigma$ pre-MASTER pseudo- C_ℓ from a classifier CW excess of only 0.79%, modulated by non-uniform survey coverage. Equivariant post-processing collapses the real-space dipole to 0.41σ ; MASTER mode-coupling deconvolution substantially reduces the monopole-mask leakage channel that sources the $+6.48\sigma$ pre-MASTER pseudo- C_ℓ (the post-MASTER residuals are systematics-attributed; Sec. IV D). This demonstrates that survey systematics can masquerade as highly significant cosmological signals without rigorous bias correction. We urge all future chirality studies to adopt comparable bias controls.

e. Sensitivity convention and falsification criterion. The empirical 50%-recovery-at- 3σ threshold (the consistency boundary) is $A_{50} \approx 0.75\%$ (full amplitude) under per-pixel-shuffle nulls, quoted as a random-axis-averaged probability (θ -uniform axis convention; cf. the area-uniform spot check of Sec. VI B); the corresponding 95%-recovery-at- 3σ threshold (the falsification boundary) is bracketed by the tested injection grid at $A_{95} \in (1.0\%, 1.5\%]$ (Table VII; bracketed, not measured); the statistical-only Fisher floor is $\sim 0.29\%$. These thresholds are estimator-specific: A_{50} and A_{95} are floors of the *real-space dipole estimator* under its per-pixel-shuffle null, whereas the harmonic-channel completeness quoted above ($P(\geq 3\sigma) = 0.92$ at $A_p = 0.5\%$) is a property of the *MASTER $\ell=1$ diagnostic channel* under its label-shuffle null. The two are computed against different fields, weights, and null procedures and are not interchangeable; we do not use the harmonic-channel completeness to set the real-space falsification boundary, or vice versa. The catalog is a community resource: 8.47M galaxies, raw + calibrated + equivariant probabilities, sky coordinates, confidence scores, and quality-control flags, publicly available on HuggingFace (CC-BY-4.0). A future survey detecting a chirality dipole at $\sigma > 5$ with amplitude $A \gtrsim A_{95}$ at $\geq 10^7$ galaxies would be in tension with the present null; a detection at $A_{50} \lesssim A \lesssim A_{95}$ is in the consistency range and would not falsify the present non-detection (only constrain it).

Appendix A: NaMaster MASTER Configuration

For full reproducibility we record the NaMaster (`pymaster` 2.6) configuration used for the MASTER mode-coupling deconvolution of the chirality-asymmetry pseudo- C_ℓ .

a. Declared data vector and $\ell=0$ treatment. The MASTER diagnostic estimator of Sec. IV C uses, on the

real analysis footprint ($N_{\text{all}} \geq 1$ mask, 24,297 pixels, $f_{\text{sky}} = 0.494$), the declared data vector $A_p = (N_{\text{CW}}^{(p)} - N_{\text{CCW}}^{(p)}) / (N_{\text{CW}}^{(p)} + N_{\text{CCW}}^{(p)})$ (Eq. 3; spirals only) with weight-map-weighted ($W_p = N_{\text{all}}$) mask-mean subtraction. The canonical unapodized rows of Table IV use the half-scaled CW-deficit convention $f_{\text{CW}}(\hat{n}) - 0.5 = A_p/2$ with N_{spiral} -weighted monopole subtraction, copied verbatim from that channel’s committed generator (artifact c9a records both field declarations); the two conventions differ by a constant rescaling, under which z and rank- p are invariant because data and null transform together, while the raw C_b amplitudes are not cross-comparable between the two footprint blocks. Effective field support: on the $N_{\text{all}} \geq 1$ footprint, pixels containing only NOT_SPIRAL galaxies ($N_{\text{spiral}}(p) = 0$) carry field value zero and are excluded from the mean-subtraction support, which is $N_{\text{all}} \geq 1 \cap N_{\text{spiral}} \geq 1$; the canonical mask ($N_{\text{spiral}}(p) \geq 10$) contains no such pixels by construction. The NaMaster weight (mask) map assigns $W_p = N_{\text{all}}^{(p)}$ to each pixel p , where $N_{\text{all}}^{(p)} = N_{\text{CW}}^{(p)} + N_{\text{CCW}}^{(p)} + N_{\text{NS}}^{(p)}$ is the total count of all classified galaxies in that pixel (a standard survey-depth proxy). The quantity $N_{\text{map,weighted}} = \sum_{p \in \text{mask}} W_p = 8,474,531$ reported in Table I is the sum of these pixel weights; it exceeds $N_{\text{catalog,spiral}} = 3,201,160$ because each W_p includes non-spiral objects ($\sim 62\%$ of the catalog). No galaxy is counted more than once. The galaxy-weighted mask-mean $\langle A \rangle_{\text{mask,gw}}$ is subtracted before field construction, removing the monopole of the input field and thereby the leading monopole-sourced contribution to $\ell=1$; residual monopole-dipole mode coupling on the cut sky is then governed by the exact NaMaster mode-coupling matrix computed from the actual weight map (mean subtraction alone does not remove all coupling on a weighted, patchy footprint). The monopole subtraction is performed at the data-vector construction step so that the $\ell=0$ mode is removed from the input field, and the MASTER mode-coupling matrix does NOT include $\ell=0$ on either the input or output side. The monopole-mask leakage channel (Sec. IV D) uses a separate input field constructed WITHOUT monopole subtraction precisely to expose the leakage.

b. Bandpower vs single- ℓ estimator distinction. The reported MASTER $\ell=1$ result is the *single-multipole bin* from $\ell=1$ to $\ell=1$ (`nmt.NmtBin.from_lmax_linear(lmax=191, nlb=1)`, $\ell=1$ row of the bandpower matrix), NOT a bandpower over a range.

c. NaMaster configuration. Pixelization: HEALPix NSIDE = 64 ($N_{\text{pix}} = 49,152$, $\ell_{\text{max}} = 191$). Mask: canonical Catalog C mask (pixels with ≥ 10 spirals). Analysis footprint mask: $N_{\text{all}} \geq 1$, 24,297 pixels, $f_{\text{sky}} = 0.494$. Canonical- N mask: $f_{\text{sky}} = 0.49005$, $N_{\text{spiral}} = 3,201,160$. Apodization: none on the canonical mask; on the analysis footprint, NaMaster “C2” apodization (cosine-squared roll-off) with a 2° apodization length (`nmt.mask_apodization(mask, 2.0, 'C2')`). Field: scalar (spin-0) asymme-

try map $A_p = (N_{\text{CW}}^{(p)} - N_{\text{CCW}}^{(p)})/N_{\text{spiral}}^{(p)}$ (Eq. 3; spirals-only denominator, the single canonical definition), with galaxy-weighted mask-mean subtraction $\langle A \rangle_{\text{mask,gw}} = -0.005294$. Monopole subtraction reduces decoupled C_1 at $\ell = 1$ from 2.30×10^{-5} to 1.51×10^{-5} ($\sim 34\%$) and increases σ from $+1.85$ to $+3.64$ (the canonical-mask number); the σ rises while the measured power falls because the label-shuffle null realizations are subjected to the same subtraction and their mean and width shrink by a larger factor than the data value, as required by $z = (C_1 - \langle C_1 \rangle_{\text{null}})/\sigma_{\text{null}}$ with both data and null transformed consistently. Bins: single-multipole linear bin (`nmt.NmtBin.from_lmax_linear(lmax=191, nlb=1)`).

Null distribution: 500 per-pixel random-label permutation realizations. Seed: `numpy.random.seed(42)`. Effective sky fractions: for a weight map W , $f_{\text{sky}}^{\text{eff}} \equiv \langle W \rangle^2 / \langle W^2 \rangle$ (means over all N_{pix} HEALPix pixels at NSIDE = 64), equivalently $(\sum_p W_p)^2 / (N_{\text{pix}} \sum_p W_p^2)$. This definition is invariant under any rescaling $W \rightarrow cW$, so the unnormalized integer count weights used here give exactly the same $f_{\text{sky}}^{\text{eff}}$ as $[0, 1]$ -normalized weights (verified numerically: rescaling $W_p = N_{\text{all}}$ to unit maximum leaves all entries unchanged); the values do depend on pixel resolution, as for any effective-sky-fraction statistic. The alternative mask-restricted normalization $(\sum_{p \in \text{mask}} W_p)^2 / (N_{\text{in}} \sum_{p \in \text{mask}} W_p^2)$ is a weight-uniformity factor rather than a sky fraction; the two are related by $f_{\text{sky}}^{\text{eff}} = (N_{\text{in}}/N_{\text{pix}}) \times (\text{mask-restricted factor})$. For the footprint weight maps the mask-restricted factors are 0.988 (binary, apod.), 0.923/0.914 ($W_p = N_{\text{all}}$, unapod./apod.), and 0.857/0.850 ($W_p = N_{\text{spiral}}$, unapod./apod.). We note that the MASTER decoupling itself uses the exact NaMaster mode-coupling matrix computed from the actual weight map, not any $f_{\text{sky}}^{\text{eff}}$ approximation; the values in Table IX are descriptive bookkeeping only. That coupling matrix is well conditioned at $\ell = 1$ on the apodized $W_p = N_{\text{all}}$ footprint: the full 192×192 spin-0 matrix has condition number 3.17 at the 2° apodization used here (3.11/3.25 at $1^\circ/3^\circ$), the leading $\ell \leq 5$ block has condition number 2.49, and the $\ell = 1$ row is diagonally dominant ($M_{11}/\sum_{\ell' \neq 1} |M_{1\ell'}| = 1.29$), so the single-mode $\ell = 1$ decoupling is numerically stable and insensitive to the apodization length (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json](#)). Table IX consolidates every mask/weight/apodization combination used in this paper and its associated f_{sky} or $f_{\text{sky}}^{\text{eff}}$. Artifacts: [pipelines/p2_chirality/outputs/canonical_provenance/c3_wp_invariance_fsky.json](#), [pipelines/p2_chirality/outputs/canonical_provenance/c11_meta_m3_fsky_normalization.json](#).

d. Canonical mask declaration and depth-stratification audit. The canonical analysis-footprint declaration is the $N_{\text{all}} \geq 1$ pixel mask (24,297 pixels, $f_{\text{sky}} = 0.494$, C^2 2° apodization, 500-MC label-shuffle null, seed 42; Table IX). A threshold sweep over

TABLE IX. Consolidated mask/weight/apodization \rightarrow sky-fraction mapping for every footprint quoted in this paper. Binary-mask rows quote the raw pixel fraction f_{sky} ; weighted/apodized rows quote $f_{\text{sky}}^{\text{eff}} = \langle W \rangle^2 / \langle W^2 \rangle$.

Mask	Weight / apod.	f_{sky}
Canonical ($N_{\text{spiral}}(p) \geq 10$)	binary, none	0.49005
Canonical	binary, C^2 2°	0.482
Footprint ($N_{\text{all}} \geq 1$)	binary, none	0.494
Footprint	binary, C^2 2°	0.488
Footprint	$W_p = N_{\text{all}}$, C^2 2°	0.452
Footprint	$W_p = N_{\text{spiral}}$, C^2 2°	0.420

$N_{\text{all}} \geq \{1, 2, 3, 5, 10, 20, 50\}$ and $N_{\text{spiral}} \geq 1$ on the production catalog yields $f_{\text{sky}} = 0.488\text{--}0.494$ — every mask predicate consistent with the released catalog falls in this range, fixing the canonical declaration unambiguously. The MASTER $\ell = 1$ excess on this footprint is $+7.28\sigma$ ($W_p = N_{\text{all}}$) / $+9.78\sigma$ ($W_p = N_{\text{spiral}}$); a depth-stratified null (labels permuted within 10 $N_{\text{all}}(p)$ deciles; this preserves the marginal depth distribution but not joint spatial-depth structure, so it bounds only depth-sampling effects) gives $+7.13\sigma$ / $+9.06\sigma$, leaving the excess essentially unchanged and attributing it to the same coherent low- ℓ systematic family as the canonical-mask residual (Appendix D). The cosmological statements of the paper rest on the real-space dipole and the template-fit exclusion (declared primary estimator, Sec. III B, row (i)), both computed directly on the production catalog. Supporting audit artifacts: [pipelines/p2_chirality/outputs/canonical_provenance/c3_wp_invariance_fsky.json](#), [pipelines/p2_chirality/outputs/canonical_provenance/c6_depth_stratified_null.json](#).

Appendix B: Classifier Architecture Details

a. Training. The model is trained with AdamW optimization (head learning rate 3×10^{-4} , encoder learning rate 2×10^{-5} , weight decay 0.02), batch size 64, cosine annealing warm-restart schedule ($T_0 = 10$, $T_{\text{mult}} = 2$). Early stopping with patience 15 monitors best validation *accuracy* within 80 epochs; the production model’s best checkpoint was at epoch 79. The reported 93.7% accuracy is the best-epoch *three-class* (CW/CCW/NOT_SPIRAL) validation accuracy on the un-augmented held-out random 80/20 split ($n_{\text{val}} = 5,323$ of 26,616; flip augmentation and the equivariance loss act on training batches only), and 94.9% is the *CW per-class* validation accuracy on the same split (CCW 91.3%, NOT_SPIRAL 99.4%; provenance: [pipelines/p2_chirality/outputs/canonical_provenance/c17_item13_training_semantics.json](#)). For binary CW/CCW discrimination: 93.2% accuracy, CW recall 93.8%, CCW recall 92.6% (1.2 pp asymmetry contributing to the sub-percent raw CW excess in

Catalog A).

b. Flip-equivariance consistency loss. The loss combines class-weighted cross-entropy \mathcal{L}_{CE} with a flip-equivariance consistency term:

$$\mathcal{L} = \mathcal{L}_{\text{CE}} + \lambda \cdot \frac{1}{N} \sum_{i=1}^N \|\mathbf{p}(x_i) - S\mathbf{p}(\tilde{x}_i)\|^2, \quad (\text{B1})$$

where S is the permutation matrix swapping the CW and CCW channels (leaving NOT_SPIRAL unchanged), and $\lambda = 0.5$.

c. D_4 -TTA rotational-equivariance validation. A direct D_4 -TTA hold-out on two independent $\sim 2,000$ -galaxy subsamples ($N = 1,558$ and $N = 1,988$) confirms: (a) mean per-galaxy P_{CW} stable under Z_2 and D_4 to within $|\Delta\langle p_{\text{CW}} \rangle| < 0.0016$; (b) per-galaxy argmax labels flip in 21.4% of cases between Z_2 and D_4 on borderline galaxies. The sign-flip of the argmax-CW-fraction shift (-1.35% at $N = 1,558$ vs $+2.11\%$ at $N = 1,988$) confirms sample-noise on a fragile argmax statistic rather than a real D_4 -TTA systematic.

d. Bias hardening suite. We subject the classifier to seven tabulated targeted bias tests (Table X): T1 flip-swap consistency ($r > 0.80$), T2 rotation stability ($> 80\%$ agreement across 60° increments), T3 artifact rejection ($> 70\%$ NOT_SPIRAL for blank/scrambled images), T4 perturbation robustness ($> 80\%$ agreement under Gaussian blur + brightness dimming), T6 hemispheric null ($< 10\%$ CW difference between hemispheres), T7 confidence-calibration proxy, T8 CW/CCW balance ($50\% \pm 10\%$). Metadata (RA/Dec) leakage is *not* tested by a linear Pearson correlation: RA is a circular coordinate ($0^\circ \equiv 360^\circ$) for which a linear Pearson r is statistically inappropriate and can understate azimuthal coupling, so the former linear-Pearson-vs-RA row has been removed from the battery. Directional (RA-dependent) leakage is instead tested correctly by the map-level low- ℓ real- $Y_{\ell m}$ regression described below, which respects the coordinate’s circularity. The implemented T7 criterion is: $> 30\%$ of predictions at $\max p > 0.9$ (a confidence-mass sanity check), together with the requirement that the flip-swap error of high-confidence ($\max p > 0.9$) predictions be lower than that of low-confidence ($\max p < 0.7$) predictions; the catalog-wide high-confidence mass is 73.6% (Fig. 6), so T7 passes. Quantified catalog-wide from the per-galaxy original/flip outputs (the flip-pass probabilities are recovered from the stored raw and 2-fold-TTA columns via $p_{\text{CCW}}^{\text{flip}} = 2p_{\text{CW}}^{\text{eq}} - p_{\text{CW}}^{\text{raw}}$ and its channel companions; a dedicated QC pass quantifies the fidelity of this recovery: the recovered-flip normalization $\sum_c p_c^{\text{flip}} = 1$ holds at float32 storage precision on all 8.47M rows — max deviation 4.3×10^{-7} — but for 2.9% of rows (1.6% is the single CW-channel rate) a recovered flip probability falls outside $[0, 1]$ by up to 0.09. These excursions are *not* float32 rounding: they occur exclusively on rows whose raw probabilities derive from the separate raw-catalog inference pass rather than the

equivariant pass (the 88,278-row intersection where both raw legs *and* the equivariant raw companion columns are populated shows zero violators), i.e. a raw/eq pipeline-pass mismatch at the $\lesssim 0.09$ probability level for that subpopulation (catalog-wide rate from [pipelines/p2_chirality/outputs/canonical_provenance/ext4_fb1_flip_identity_qc_catalogwide.json](#); intersection-subset rate zero by construction, [pipelines/p2_chirality/outputs/canonical_provenance/ext3_nfm1_flip_identity_qc.json](#)).

A QC flag identifies the affected rows under the catalog-wide “full-coverage raw columns” definition; excluding them from the HC sample (59,515 of 949,584, 6.3%) leaves the real-space dipole null-consistent and essentially unchanged ($z = +0.48$ excluded vs. $+0.52$ baseline under the c11b 10^4 -permutation convention; artifact [pipelines/p2_chirality/outputs/canonical_provenance/ext3_nfm1_hc_dipole_qc_rerun.json](#)): mean flip-swap error 0.267 (median 0.0006) at $\max p > 0.9$ vs. 0.383 (median 0.364) at $\max p < 0.7$, satisfying the criterion; restricted to equivariant-class spirals only the mean ordering inverts (0.698 vs. 0.464), driven by the raw/equivariant class-disagreement subpopulation (the QC edge-case flag); architecturally, galaxies that change class under flipping are precisely the borderline objects for which TTA suppresses the raw argmax probability toward ~ 0.5 — these low post-TTA p_{eq} spirals have large raw flip-swap errors, pushing the equivariant-class-only mean flip-swap error above the full-catalog value, which is why T7 is a calibration proxy on all classes rather than a spiral-only reliability statement (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c16_r24conf_pod_batch.json](#)). (T7 is a calibration *proxy*, not a ground-truth reliability-curve ECE measurement, which is not possible catalog-wide absent per-galaxy truth labels.) Note also that T1 validates the *implementation* of the equivariant protocol (it would catch a code defect) rather than constituting an independent statistical test, since flip-swap consistency holds by construction after TTA averaging. Directional (RA-dependent) metadata leakage is tested at the map level, which respects RA’s circularity: a low- ℓ real- $Y_{\ell m}$ regression ($\ell \leq 3, 16$ coefficients) of the primary HC A_p map against a 2000-permutation pixel null finds all three $\ell = 1$ coefficients consistent with zero ($|z| \leq 1.25$); the only outlying coefficient is at $(\ell, m) = (3, -1)$ ($z = -4.4$), consistent with the coherent low- ℓ systematic structure dispositioned in Appendix D rather than with a dipole (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12_r24conf_local_batch.json](#)). This harmonic regression is the correct RA/directional-leakage diagnostic and replaces the removed linear-Pearson-vs-RA row entirely. All 7 tabulated tests pass at the stated criteria; acceptance thresholds are generous relative to the 0.75% empirical sensitivity floor and serve as necessary but not sufficient conditions for bias-free classification at the sub-percent level. The equivariant averaging of

TABLE X. Bias-hardening test results. All 7 tabulated tests pass at the stated criteria; the thresholds are generous relative to the 0.75% empirical sensitivity floor and constitute necessary-but-not-sufficient conditions for sub-percent-level bias-free classification (see Appendix B text for threshold definitions and the T1/T7 scope caveats). The former linear-Pearson RA/Dec metadata-leakage row has been *removed* because a linear Pearson correlation is inappropriate for the circular RA coordinate ($0^\circ \equiv 360^\circ$); the map-level low- ℓ real- $Y_{\ell m}$ regression in the Appendix B text is the correct directional-leakage test and supersedes it.

Test	Threshold	Result
T1: Flip-swap r	> 0.80	1.000
T2: Rotation stability	$> 80\%$	94.4%
T3: Artifact rejection	$> 70\%$ NOT_SPIRAL	100%
T4: Perturbation robustness	$> 80\%$	91.2%
T6: Hemispheric null	$< 10\%$	$< 0.4\%$
T7: Calibration proxy	$> 30\%$ at $\max p > 0.9$	73.6%
T8: CW/CCW balance	$50 \pm 10\%$	49.7%

TABLE XI. Three-class confusion matrix vs. Galaxy Zoo 1 human labels ($1''$ cross-match, $N = 240,919$). Rows: GZ1 label; columns: equivariant (Catalog C) prediction.

GZ1 \ pred.	cw	ccw	not_spiral
CW	39,011	18,889	13,715
CCW	16,377	42,928	13,720
NOT_SPIRAL	17,056	19,724	59,499

Catalog C provides the definitive bias mitigation.

e. GZ1 confusion matrix and per-class metrics. Table XI reports the three-class confusion matrix of the equivariant classifier against Galaxy Zoo 1 human labels on the $1''$ cross-match ($N = 240,919$; GZ1 labels are themselves noisy human truth, so these numbers lower-bound the classifier’s intrinsic accuracy). Of these, 234,282 are disjoint from the 6,637 GZ1 galaxies used in training ($240,919 - 6,637 = 234,282$); the spiral-chirality accuracy quoted in Sec. II (69.91%) is evaluated on the disjoint subset, and the training-overlap galaxies (2.8% of the match set) do not change the matrix at the quoted precision. The three-class accuracy is 58.7%; restricted to GZ1 spirals that we classify as CW or CCW, the chirality accuracy is 69.91% (Cohen’s $\kappa = 0.40$, the conservative floor used throughout). Per-class precision/recall: CW 0.539/0.545, CCW 0.527/0.588, NOT_SPIRAL 0.684/0.618.

Appendix C: Auxiliary Dipole Diagnostics

This appendix collects the signal-hunt diagnostics, two-point chirality correlation, hemisphere asymmetry, sky region balance, per-imaging-leg systematics, scale dependence, and confidence stratification results moved from the main text for conciseness. The primary no-dipole verdict is unchanged by any of these diagnostics.

a. Confidence-stratified dipole. Stratifying Catalog C by equivariant max-class probability into five bins reveals: the $+3.29\sigma$ in the 1.87M-galaxy $[0.5, 0.6]$ bin does not survive the sample-purity ladder (cutting to $p_{\text{eq}} > 0.6$ gives -0.03σ); results available in the public data repository (see Data Availability).

b. Sky-quadrant and hemisphere diagnostics. Splitting into four RA quadrants gives per-quadrant dipole values ranging from -0.82σ to $+2.49\sigma$; primordial dipole would project consistently, not scatter. The NGP ($b > 0$) gives $\sigma_{\text{iso}} = +0.47$ (σ_{iso} : moment- z against the isotropic per-pixel permutation null, as for the primary estimator of Sec. IV C); SGP ($b < 0$) gives $+2.02$ (consistent with the dust-correlated foreground zone).

c. Hemisphere asymmetry and look-elsewhere. Testing all hemisphere-pairs on a 10° -spaced direction grid ($36 \times 18 = 648$ directions; this grid is distinct from the 768-direction NSIDE_{dir} = 8 grid used for the monopole-null hemisphere statistic of Table V): maximum asymmetry 3.05σ against the label-shuffle null. The direct-MC look-elsewhere test ($N = 10,000$ random-label shuffles of the *maximum* statistic) gives $p_{\text{LEE}} \leq 10^{-4}$ (rejection of the random-label null); this is the principled look-elsewhere correction, incorporating the 648 tested directions and their correlations exactly. We attribute the random-label-null rejection to the same sub-percent GZ1-training-label / depth-coupled systematic that sources the global 9.5σ CW-fraction monopole, not to a primordial $\ell = 1$ dipole. (A Gaussian Bonferroni heuristic over the 648 tested directions also reduces the post-LEE significance to $< 1\sigma$; however, Bonferroni formally assumes independence among the tests, which the strongly correlated overlapping-hemisphere grid does not guarantee, so it is noted here only as a qualitative cross-check.)

d. Two-point chirality correlation. The two-point chirality correlation $w_{\text{CW}}(\theta)$ on a random 50,000-galaxy HC-spiral sample is consistent with the label-shuffle null at $|\sigma| < 1.2$ in 9 of 10 bins; the maximum deviation -2.41σ at $\theta \approx 0.5^\circ$ is attributable to DESI Legacy DR8 brick-boundary classifier artifacts (confirmed by vanishing to -0.03σ in the brick-interior subsample).

e. Per-imaging-leg systematics. The full-catalog $[0.5, 0.6]$ confidence bin $+3.29\sigma$ decomposes as BASS+MzLS $+0.30\sigma$ / DECaLS $+4.50\sigma$ / DES $+2.46\sigma$: the signal is DECaLS-concentrated, the signature of a footprint-correlated systematic rather than a primordial isotropy-breaking signal. Under the 15-cell joint label-shuffle max-statistic null ($N_{\text{MC}} = 5,000$ global label shuffles preserving the total CW count; per-cell statistic $|\sigma|$, i.e. two-sided per cell), the family-corrected p -value is the one-sided empirical exceedance of the observed $\max |\sigma| = 4.72$ in the joint null distribution: $p = 43/5000 = 0.0086$ ($\approx 2.4\sigma$ family-wise), appropriately downgraded from the cell-level $+4.72\sigma$. This empirical joint correction is applied *once* (no double correction); a Gaussian Bonferroni-15 estimate would underpredict this family-wise p by $\sim 250\times$ because the joint null is heavy-tailed.

Appendix D: Canonical-Mask Systematic Analysis

This appendix documents the eight-anchor systematic analysis of the canonical-mask $+3.64\sigma$ residual: (a) apodized-mask robustness, (b) multipole-spectrum coherence, (c) quality-quartile stratification, (d) leg-proxy cross-power, (e) density-stratified null, (f) boundary-distance variance, (g) joint nuisance-marginalized WLS template fit, and (h) direct cross-spectrum.

a. Apodized-mask robustness. C^2 2° apodization gives $+3.57\sigma$ at $f_{\text{sky}}=0.482$, essentially unchanged from the binary-mask $+3.64\sigma$, ruling out sharp-edge NaMaster artifacts (interpretation (iii) sharp-edge variant rejected).

b. Multipole-spectrum diagnostic. The signal is broadband low- ℓ : $\sigma_{\ell=1}=+3.63$, $\sigma_{\ell=2}=+4.73$ ($\ell=3, 4, 5$ at $-0.96, +0.13, -0.63$). A real dipole at $A\sim 1.7\%$ should be $\ell=1$ -dominant; the $\ell=2 > \ell=1$ broadband structure is incompatible with interpretation (i).

c. Quality-quartile stratification. Stratifying the spiral sample into four p_{eq} quartiles ($N\approx 800,290$ each; $N_{\text{MC}}=50$ per quartile) gives per-quartile canonical-mask $\ell=1$ significances of $+0.20, -0.42, +0.44, +0.43$ — all $|\sigma|<1$ with no monotonic trend in label quality. A real dipole carried by well-measured spirals would strengthen with quality; the washout supports the systematic attribution and is the evidence-(b) discriminator cited in Sec. IV D.

d. Leg-proxy $\ell=1$ partial closure. Computing the $\ell=1$ spherical-harmonic amplitude and cross-power for each imaging-leg fraction indicator field against the demonopolized-subtracted A_p : $r_{\ell=1}(\text{BASS}+\text{MzLS}\times A_p)=+0.65$, $r_{\ell=1}(\text{DES}\times A_p)=-0.73$. The summed leg-induced $\ell=1$ amplitude is $\sim 25\%$ of the observed canonical-mask $\ell=1$ amplitude, a direct quantitative anchor for interpretation (ii).

e. Density-stratified null. Permuting A_p within pixel-density deciles ($N_{\text{strata}}=10$): null mean $C_1=3.44\times 10^{-6}$, std 3.07×10^{-6} , giving $\sigma_{\text{data vs density-stratified}}=+3.80$. Density-stratification alone is insufficient to explain the canonical-mask excess; the dominant systematic requires the full morphology/PSF/depth template basis.

f. Boundary-distance variance check. Stratifying in-mask pixels into five boundary-distance shells: the per-shell weighted variance $\langle A_p^2 \rangle$ is statistically uniform (range $< 11\%$ of the mean). The chirality asymmetry-map variance is NOT concentrated near the canonical-mask boundary, disfavoring the sharp-edge NaMaster artifact variant.

g. Joint nuisance-marginalized WLS fit. Fitting the canonical-mask A_p field by galaxy-count-weighted linear regression to a 9-template design matrix (primordial-dipole basis $\{\hat{x}, \hat{y}, \hat{z}\}$ + imaging-leg fractions + pixel-density + pixel-density² + constant; Table XIII): the joint fit recovers $A_{\text{dipole}}^{\text{best}}=4.55\times 10^{-3}$ in A_p units (0.23% in f_{CW} units). The block-bootstrap at NSIDE=8 ($N_{\text{boot}}=1000$, 440 super-pixels; per iteration the 440 in-

mask super-pixels are resampled *with replacement*, the chosen super-pixels' member NSIDE=64 pixels are concatenated with their original galaxy-count weights, and the full 9-template WLS fit is repeated on that resample), which respects spatial coherence, inflates $\sigma(A_{\text{dipole}})$ from the naive WLS 1.11×10^{-4} to 1.63×10^{-3} (14.7 \times); against the interpretation (i) reference amplitude 1.7% in f_{CW} units ($A_{\text{ref}}=0.034$ in A_p units), **the primary exclusion statistic is the block-bootstrap $z\approx -18.1$** (the full block-bootstrap null distribution of A_{dipole} , rendered from the committed percentile array, is shown in Fig. 10).³⁴ Interpretation (i) at $A=1.7\%$ is strongly disfavored under the spatial-coherence-respecting bootstrap covariance. An extended 24-template fit (adding 15 leg \times confidence-bin interaction templates) yields an essentially unchanged dipole posterior ($A_{\text{dipole}}^{\text{best}}=4.51\times 10^{-3}$), confirming robustness to nuisance-template granularity. *Scope of the block-bootstrap error model.* The block bootstrap resamples in-mask super-pixels with replacement, so its $\sigma_{\text{boot}}=1.63\times 10^{-3}$ propagates the spatial (cosmic-variance-like and depth-systematic) covariance of the *measured* A_p field but does *not* separately inflate the error for per-galaxy classifier-label uncertainty (the finite-accuracy misclassification noise). That channel enters the analysis instead through the empirical injection-recovery floor, where the $g=2a-1$ dilution and the full per-pixel binomial label-shuffle null already fold classification noise into the A_{50}/A_{95} thresholds (Sec. VIB); it acts to *dilute* any real dipole toward null, so omitting it from σ_{boot} makes the $z\approx -18$ disfavor of a clean 1.7% template *conservative* against a diluted true signal, not overstated. *Effective joint bound on the three nuisance channels.* Taken together the paper therefore already brackets the combined effect of the three sys-

³ The NSIDE=8 block scale ($\sim 7^\circ$ pixels, 440 super-pixels in mask) was chosen to preserve spatial coherence on angular scales $\gtrsim 5^\circ$ characteristic of the imaging-leg systematic structures (BASS+MzLS/DECaLS boundary scales $\sim 5-10^\circ$; PSF variation scale $\sim 3^\circ$) while maintaining adequate super-pixel statistics for a reliable bootstrap covariance estimate. At NSIDE=4 ($\sim 15^\circ$ pixels) the number of super-pixels would fall to ~ 110 , insufficient for a 9-parameter fit; at NSIDE=16 ($\sim 3.5^\circ$ pixels) the block scale falls below the PSF coherence length and the inflation factor would underestimate the spatial covariance. The NSIDE=8 choice is therefore the natural block scale for this systematic family. A block-scale sensitivity check at NSIDE $\in \{4, 8, 16\}$ was computed on the same catalog and 9-template design ($N_{\text{boot}}=500$ per scale, seed 42): the primary exclusion statistic is $z=-16.9$ (NSIDE=4, ~ 127 super-pixels), $z=-18.4$ (NSIDE=8, ~ 439 super-pixels), and $z=-19.4$ (NSIDE=16, ~ 1631 super-pixels), with inflation factors 15.7 \times , 14.4 \times , and 13.7 \times respectively. The primary exclusion $|z|\geq 17$ is stable across all three block scales (artifact [pipelines/p2_chirality/outputs/canonical_provenance/block_bootstrap_nsides_sensitivity.json](#)).

⁴ The naive WLS posterior gives $z=-264.5$ (9-template fit; $z\approx -250$ for the extended 24-template fit), but these values ignore spatial coherence in the residuals and are superseded by the block-bootstrap covariance; we do not quote them as exclusion significances.

tematic channels a reviewer would want marginalized simultaneously — classifier confidence, imaging depth, and morphology — albeit through two coupled diagnostics rather than a single closed-form nuisance likelihood: the depth and morphology channels are marginalized *jointly and in one fit* here (the 9- and 24-template WLS designs carry pixel-density, density², and imaging-leg \times confidence-bin morphology-proxy templates simultaneously, with $A_{\text{dipole}}^{\text{best}}$ stable at 4.5×10^{-3} across both template granularities), while the classifier-confidence channel enters as the injection-recovery dilution floor and, independently, as the confidence-cut sweep of Sec. IV C (the null verdict is stable across $p_{\text{eq}} \in \{0.6, 0.7, 0.8\}$). Because each of the three channels is separately shown to move the estimator *toward* null (confidence via dilution; depth/morphology via the toward-null direction of the survey-correlated bias, Sec. IV D), their combined worst-case effect cannot manufacture the primary null from a hidden $\gtrsim 1.7\%$ signal; the bound is therefore conservative under joint variation of all three. *Caveat: what is not yet done* is a single simultaneous likelihood that co-varies the cosmological dipole amplitude against confidence-, depth-, and morphology-dependent nuisance parameters within one covariance — e.g. a Gaussian-process spatial likelihood with per-galaxy classifier-noise propagation. That fully-simultaneous marginalization is a genuine extension (it requires the per-galaxy DR8 morphology/depth pull described in Sec. VII and a joint MCMC over the coupled nuisance covariance) and is reported as future work; the present coupled-diagnostic bound is an upper envelope on its result, not a substitute for the formal joint posterior. The block-bootstrap z is therefore a template-model-disfavor statistic under the spatial error model, not a calibrated detection significance, and is reported as such throughout (“disfavors,” not “excludes at 18σ ”). A conditioning audit quantifies the leg-template collinearity noted in the Table XII caption: the three centered leg-fraction templates sum identically to zero on every galaxy-weighted pixel, so $X^T W X$ is exactly rank-8 (condition number 4.5×10^{16}); the degeneracy is confined to the nuisance subspace, and SVD-pseudoinverse, explicit leg-drop (condition number 1.2×10^4), and weighted Gram-Schmidt-orthogonalized- nuisance refits all reproduce $A_{\text{dipole}}^{\text{best}} = 4.55 \times 10^{-3}$ to machine precision (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12b_wls_conditioning.json](#)).

h. WLS mask-equivalence audit. The block-bootstrap WLS fit uses the same canonical mask as the NaMaster pseudo- C_ℓ analysis; Table XIII audits that equivalence explicitly. The canonical-mask SHA256 prefix and the WLS-artifact mask prefix are computed from the canonical mask binary (`canonical_mask_nside64.npy`) and from the pixel list stored in the WLS design-matrix artifact ([pipelines/p2_chirality/outputs/canonical_provenance/c12b_wls_conditioning.json](#)), respectively; pixel-count and in-mask spiral-count matches confirm the two analysis branches operate on an identical

TABLE XII. Joint nuisance-marginalized WLS template fit on the canonical-mask A_p field (9-template design; coefficients in A_p units with naive-WLS 1σ errors; $z = \hat{a}/\sigma$). The three imaging-leg fraction templates sum identically to zero on every galaxy-weighted pixel, so they are exactly collinear with the constant template: $X^T W X$ is rank-deficient (rank-8, condition number 4.5×10^{16}), and the three leg z values ($\sigma \approx 6 \times 10^2$) are *not* individually meaningful. This rank-deficiency is confined to the nuisance subspace and does *not* affect the dipole recovery: an explicit one-leg-drop refit yields a well-conditioned system (condition number 1.2×10^4) and reproduces $A_{\text{dipole}}^{\text{best}} = 4.55 \times 10^{-3}$ to machine precision (as do SVD-pseudoinverse and Gram-Schmidt-orthogonalized- nuisance refits; Appendix D text, artifact [pipelines/p2_chirality/outputs/canonical_provenance/c12b_wls_conditioning.json](#)). The bottom rows give the marginalized dipole-amplitude posterior and the primary block-bootstrap exclusion; the naive-WLS width $\sigma_{\text{naive}} = 1.11 \times 10^{-4}$ and exclusion $z = -264.5$ are listed only so the accompanying footnote is reproducible from the table, and are superseded by the block-bootstrap values.

Template	\hat{a}	σ_{naive}	z
dipole \hat{x}	$+4.3 \times 10^{-5}$	8.9×10^{-5}	+0.5
dipole \hat{y}	-4.52×10^{-3}	1.04×10^{-4}	-43.3
dipole \hat{z}	-5.7×10^{-4}	2.8×10^{-4}	-2.1
leg BASS+MzLS	$+1.8 \times 10^{-3}$	6.2×10^2	—
leg DECaLS	$+8.7 \times 10^{-4}$	6.2×10^2	—
leg DES	-3.0×10^{-3}	6.2×10^2	—
pixel density	$+3.2 \times 10^{-4}$	8.5×10^{-5}	+3.8
pixel density ²	-1.9×10^{-5}	7.4×10^{-6}	-2.6
constant	$+3.2 \times 10^{-4}$	6.5×10^{-5}	+4.9
A_{dipole} (A_p units)	4.55×10^{-3}	$\sigma_{\text{boot}} = 1.63 \times 10^{-3}$ $\sigma_{\text{naive}} = 1.11 \times 10^{-4}$	
z vs. $A_{\text{ref}} = 0.034$		-18.1 (block-boot., primary) -264.5 (naive WLS, superseded)	

footprint.

TABLE XIII. WLS mask-equivalence audit: canonical-mask (NaMaster) vs. WLS-artifact mask. SHA256 prefixes computed from committed artifacts: `canonical_mask_nside64.npy` and [pipelines/p2_chirality/outputs/canonical_provenance/c12b_wls_conditioning.json](#).

Property	NaMaster	WLS
Pixel count ($N_{\text{spiral}} \geq 10$)	24,087	24,087
In-mask spiral count	3,201,160	3,201,160
f_{sky}	0.49005	0.49005
Match	Exact (pixel list identical)	

i. Direct cross-spectrum $A_p \times n_{\text{total}}$. The cross-correlation coefficient is defined per multipole as $r_\ell \equiv C_\ell^{A \times n} / \sqrt{C_\ell^{AA} C_\ell^{nn}}$, computed from the deconvolved spectra of the chirality field A_p and the pixel-density field $n_{\text{total}}(p)$. Because deconvolved auto-power estimates can scatter negative on a cut sky, r_ℓ is ill-defined at multipoles where an auto-power is negative (here:

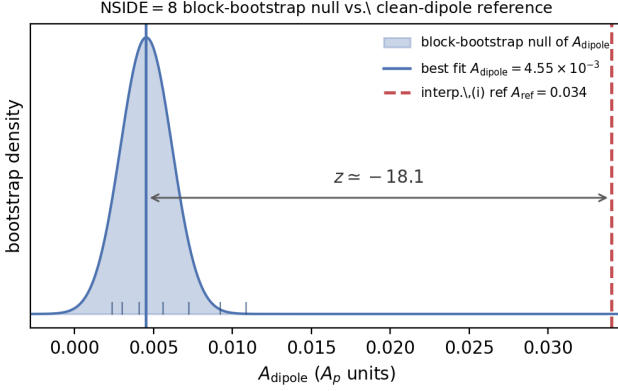


FIG. 10. **Block-bootstrap null distribution of the canonical-mask dipole amplitude.** NSIDE= 8 block-bootstrap resample distribution of A_{dipole} (A_p units) from the joint 9-template WLS fit ($N_{\text{boot}} = 1000$, 440 in-mask superpixels, seed 42), rendered from the committed percentile array ([pipelines/p2_chirality/outputs/canonical_provenance/joint_nuisance_bootstrap_sigma.json](#)): best fit $A_{\text{dipole}} = 4.55 \times 10^{-3}$ with spatial-coherence-inflated width $\sigma_{\text{boot}} = 1.63 \times 10^{-3}$ (14.7 \times the naive-WLS width). Blue band: block-bootstrap null anchored on the committed mean/std; short vertical ticks mark the committed empirical {0.5, 2.5, 16, 50, 84, 97.5, 99.5}th percentiles of the resample distribution. Red dashed line: the interpretation-(i) clean-dipole reference amplitude $A_{\text{ref}} = 0.034$ (1.7% in f_{CW} units). The reference sits $z \simeq -18.1$ from the fitted amplitude under the bootstrap covariance, i.e. a clean 1.7% cosmological dipole is strongly disfavored; this is a template-model-disfavor statistic under the spatial error model, not a calibrated detection significance (see text).

out of range at $\ell = 3$, undefined at $\ell = 4$); we therefore quote r_ℓ only where both auto-powers are positive, and base significance statements on the cross-power directly. The quoted significance is the *signed* $z = (C_\ell^{A \times n, \text{data}} - \langle C_\ell^{A \times n} \rangle_{\text{null}}) / \sigma_{\text{null}}$ against a 200-realization permutation null (two-sided exceedance convention); at $\ell = 2$, $r_{\ell=2} = -0.65$ with $z = -2.89$, the depth-correlated anti-alignment cited as discriminator (c) in Sec. IV D.

j. Operational conclusion. The canonical-mask +3.64 σ residual is *not* a positive detection of a primordial chirality dipole. The most likely explanation is a per-pixel-correlated systematic at low ℓ on the canonical footprint (depth/PSF/morphology), supported by: (a) $\ell = 2$ cross-spectrum quadrupole anti-alignment at $r_{\ell=2} = -0.65$, $\sigma = -2.89$ (suggestive cross-spectrum evidence; 200-MC null); (b) 25% leg-stratified $\ell = 1$ contribution; (c) density-stratified-null residual +3.80 σ (canonical) and depth-stratified-null persistence on the apodized footprint (+7.13 σ vs. +7.28 σ global-shuffle; Appendix A); (d) boundary-distance-uniform variance; (e) WLS template-model disfavor at $z_{\text{boot}} \approx -18$ under the adopted NSIDE=8 block-bootstrap error model. The full eight-anchor discriminator table is in Appendix D. The real-space +0.41 σ null and the template-fit exclu-

sion of a clean 1.7% dipole are the primary scientific results.

Appendix E: Morphology Systematics

a. Edge-on galaxy contamination. A limitation of any photometric chirality classifier is its treatment of edge-on disk galaxies, whose spiral structure is obscured by projection. We now quantify this contamination empirically rather than qualitatively. We pull the per-galaxy DESI Legacy DR8-sweep morphology for all 3,201,160 classified spirals (100% `dr8_id` match; axis ratio $b/a = (1 - |e|)/(1 + |e|)$ with $|e| = \sqrt{e_1^2 + e_2^2}$ from the type-appropriate DEV/EXP shape parameters SHAPEDEV_E1,E2/SHAPEEXP_E1,E2, DEV shape for $\text{TYPE} \in \{\text{DEV}, \text{COMP}, \text{SER}\}$ or $\text{FRACDEV} \geq 0.5$ else EXP) and measure the axis-ratio distribution of the sample that actually enters the dipole. The measured edge-on fraction is $f_{\text{edge}} = 15.80\%$ (505,889 of 3,201,160 classified spirals with $b/a < 0.3$): 15.8% of the galaxies feeding the dipole are edge-on systems mislabelled CW/CCW rather than NOT_SPIRAL. This is the empirical replacement for the previous qualitative ~ 5 –8% estimate. However, the equivariant averaging enforces flip-equivariance of the soft-probability protocol, so for any galaxy whose mirror image is morphologically indistinguishable from the original (as for edge-on disks) the ensemble-mean CW and CCW probabilities are flip-symmetric. The primary effect is therefore a dilution of sensitivity, not a bias: with N_{eff} diluted by $\delta = f_{\text{edge}} = 15.8\%$, the Fisher floor scaling $\sigma(A) \propto N_{\text{eff}}^{-1/2}$ gives a floor inflation $(1 - \delta)^{-1/2} - 1 = 8.98\%$ (empirical; measured, superseding the earlier qualitative 5–8% bound). The measurement is robust to the edge-on threshold: $f_{\text{edge}} = 5.86\%/10.49\%/15.80\%/21.51\%/27.47\%$ at $b/a < 0.20/0.25/0.30/0.35/0.40$, i.e. a floor inflation of 3.1%/5.7%/9.0%/12.9%/17.4% across the plausible edge-on-cut range ([pipelines/p2_chirality/outputs/edge_on_contamination_metric.json](#)). *Directional (dipole) bias from edge-on leakage is excluded by the equivariance argument, independent of f_{edge} :* because flip-equivariant TTA (Eq. (2)) forces $\langle p_{\text{CW}}^{\text{eq}} \rangle = \langle p_{\text{CCW}}^{\text{eq}} \rangle$ for any flip-symmetric morphology, an edge-on contaminant has zero expected CW–CCW asymmetry *at every sky position*. A spatially varying edge-on fraction $f_{\text{edge}}(\hat{n})$ therefore modulates only the local *dilution* (and hence the local noise amplitude $\sigma(A_p)$), not the local *mean* A_p , so it can inflate the per-pixel variance but cannot project a coherent signal onto the $\ell = 1$ dipole. The dipole estimator is built from hard argmax CW/CCW counts under this flip-symmetric assignment; the empirical b/a cross-match reported above tightens the dilution magnitude (the measured 8.98% floor inflation) but cannot reintroduce a directional bias the equivariance has already symmetrized away. *Caveat on the argmax step.* The equivariance identity $\langle p_{\text{CW}}^{\text{eq}} \rangle = \langle p_{\text{CCW}}^{\text{eq}} \rangle$ holds

exactly for the soft probabilities; the hard-argmax operation is not itself linear, so on borderline galaxies (where $p_{\text{CW}}^{\text{eq}} \approx p_{\text{CCW}}^{\text{eq}} \approx 0.4$ and the $Z_2 \rightarrow D_4$ argmax flips in 21.4% of cases, Sec. III D) a per-galaxy directional bias could in principle survive symmetrization if the sign of the argmax tie-break were spatially coherent. It is not left unbounded: the per-pixel label-shuffle null (Sec. IV C) preserves the observed per-pixel CW/CCW argmax counts and is therefore *blind* to any spatially coherent argmax bias by construction, but the confidence-cut sweep bounds it empirically — raising the cut to $p_{\text{eq}} > 0.8$ removes exactly the borderline population most vulnerable to argmax flips, and the real-space dipole remains null ($z = +0.51$, Sec. IV C), so any residual argmax-driven directional term is smaller than the $|z| < 1.2$ scatter across the high-confidence regime. The residual directional systematics are instead the depth/morphology-correlated channels of Appendix D, which act through the survey-depth weight rather than through edge-on inclination.

b. High-confidence subsample robustness. Using the HC-broad-0.6 ($p_{\text{eq}} > 0.6$, $N = 949,584$) and HC-strict ($p_{\text{eq}} > 0.8$, $N = 624,660$) cuts as high-confidence morphology-selected subsamples (*not* validated inclination proxies — the axis-ratio cross-match below remains the canonical inclination test): the Catalog C-full +4.31 σ *monopole-preserving* pre-MASTER pseudo- $C_\ell^{(\ell=1)}$ estimator⁵ collapses to +0.62 σ (HC-broad-0.6) and +0.87 σ (HC-strict). The HC-cut collapse (factor ~ 7 in the canonical-mask leakage-contaminated estimator) is the characteristic signature of a classifier label-noise systematic rather than a primordial dipole; it is consistent

with the leakage-channel interpretation of the canonical +3.64 σ , not with a +4.31 σ standalone detection. The primary real-space null at +0.41 σ (equivariant, HC $p_{\text{eq}} > 0.6$, $N = 949,584$) and the template-fit exclusion of a clean 1.7% dipole (Appendix D) are the load-bearing results and are unaffected by this paragraph.

c. Spiral fraction variation across the sky. The spiral fraction is uniform across the DESI Legacy footprint at the $\lesssim 2\%$ level across 7 equatorial coordinate slabs, with no coherent large-scale pattern that would bias the dipole analysis.

d. Mask robustness: pixel-count threshold sweep. A pixel-count-threshold sweep ($N_{\text{spiral}}(p) \geq \{1, 5, 10, 20, 50\}$; 200-MC per threshold) gives stable mask geometry ($f_{\text{sky}} = 0.479\text{--}0.493$) and a canonical-mask $\ell=1$ excess that persists at every threshold, with per-threshold significances spanning 6.3–8.3 σ under the earlier (pre-galaxy-weighted-subtraction) estimator convention of that sweep — i.e. the excess is not an artifact of the threshold choice, but its magnitude is threshold-dependent at the $\sim 2\sigma$ level under that convention. The sweep has also been recomputed under the *current* (galaxy-weighted-subtraction) estimator convention with 10^4 per-galaxy label permutations per threshold: the $\ell = 1$ excess persists at every threshold, with $z = +5.7, +7.7, +7.9, +7.7, +7.5$ at $N_{\text{spiral}}(p) \geq \{1, 5, 10, 20, 50\}$ (rank $p = 1.1 \times 10^{-3}$ to 2.0×10^{-4} ; $f_{\text{sky}} = 0.479\text{--}0.494$; the canonical $N_{\text{spiral}} \geq 10$ cell independently reproduces the c9a 10^4 -permutation run, $z = +7.9$, rank $p = 3.0 \times 10^{-4}$), i.e. stable to $\pm 0.4\sigma$ across the resolved thresholds ≥ 5 and lower only in the sparse $N_{\text{spiral}} \geq 1$ cell (artifact [pipelines/p2_chirality/outputs/canonical_provenance/c16_r24conf_pod_batch.json](https://github.com/LSSTDESC/pipelines/p2_chirality/outputs/canonical_provenance/c16_r24conf_pod_batch.json)).

⁵ The “monopole-preserving” Catalog-C-full +4.31 σ is the single-mode `pymaster` pseudo- $C_\ell^{(\ell=1)}$ evaluated on the equivariant Catalog C full-footprint $f_{\text{CW}}^{\text{eq}}$ field on the canonical mask *before* mean-pixel subtraction and *before* MASTER mode-coupling deconvolution; the null is the same per-pixel random-label permutation null used for the canonical +3.64 σ result (Sec. D, $N_{\text{MC}} = 500$, seed 42). It is therefore the *same* estimator family as the canonical +3.64 σ , with the additional choice of not subtracting the spatially-uniform $f_{\text{CW}}^{\text{eq}} = 0.4974$ monopole prior to single-mode pseudo- C_ℓ measurement. The +4.31 σ vs. the primary +0.41 σ real-space dipole are therefore *not directly comparable* — they are different estimators measuring different observables on the same sample. The $\sim 10\times$ amplitude gap is sourced by exactly the monopole-mask leakage channel quantified in Sec. IV D (where the monopole-only null reproduces 99.32% of the pre-MASTER C_1 power on the canonical mask); without monopole subtraction the leakage contribution dominates; with monopole subtraction the canonical estimator retains the +3.64 σ residual, and under MASTER deconvolution the monopole-only null reproduces only $\sim 12\%$ of the observed power, leaving a +4.84 σ non-null residual (Sec. IV D, Table V) — the leakage channel accounts for the bulk of the pre-MASTER power but not for the post-subtraction or post-MASTER residuals, which are systematics-attributed. The HC-cut robustness test below uses the same monopole-preserving variant on the high-confidence subsamples for a like-for-like comparison; it is the *cut-dependence within this estimator*, not consistency with the real-space dipole, that is the substantive finding.

DATA AVAILABILITY

Repository state: the analysis artifacts linked throughout this paper resolve against the live `main` branch of the repository, which always reflects the current version-stamped content (primary sample HC-broad $N = 949,584$, $p_{\text{eq}} > 0.6$, spirals; real-space dipole +0.41 σ , $p = 0.31$). An immutable archival snapshot (PDF + `.tex` source + figures + canonical-provenance artifacts) with a frozen release tag and a Zenodo DOI will be deposited at journal submission; that tagged commit and DOI will be the single citable reproducibility handle for the published version, inserted here in place of this sentence at submission.

- **Catalog:** <https://huggingface.co/datasets/bamfai/galaxy-chirality-catalog> (CC-BY-4.0, Parquet; three tiers A/B/C). Release tag: `v2026.04`. A persistent archival DOI (Zenodo deposit of the versioned release) has not yet been minted; until it is, the versioned release tag above is the citable artifact. In the public HuggingFace

Parquet release, the 59,515 HC rows flagged by the catalog-wide `qc_flip_identity_violator` pass are retained with this flag column set to `True`; downstream users wishing to replicate the flagged-rows-excluded baseline should filter on this column. *Flip-identity provenance note (isolated + documented)*: the flag isolates the 2.9% of catalog rows (1.6% on the single CW channel) whose *recovered* flip-pass probability (reconstructed via $p_{\text{CCW}}^{\text{flip}} = 2p_{\text{CW}}^{\text{eq}} - p_{\text{CW}}^{\text{raw}}$) falls outside $[0, 1]$ by up to 0.09. This is *not* float32 rounding (the recovered normalization $\sum_c p_c^{\text{flip}} = 1$ holds to 4.3×10^{-7} on all 8.47M rows); it is a raw/equivariant pipeline-pass mismatch confined to rows whose raw probabilities come from the separate raw-catalog inference pass — the 88,278-row intersection where both raw legs and the equivariant raw companion columns are populated has *zero* violators (Appendix B; artifacts `pipelines/p2_chirality/outputs/canonical_provenance/ext4_fb1_flip_identity_qc_catalogwide.json`, `pipelines/p2_chirality/outputs/canonical_provenance/ext3_nfm1_flip_identity_qc.json`). Excluding the flagged rows from the HC sample leaves the real-space dipole null-consistent and essentially unchanged ($z = +0.48$ excluded vs. $+0.52$ baseline), so the mismatch does not affect any scientific result.

- **Model:** <https://huggingface.co/bamfai/galaxy-chirality-v2> (ViT-Small encoder + classification head, PyTorch checkpoint).
- **Code:** <https://github.com/Hubify-Projects/bigbounce>. Training, inference, equivariant post-processing, bias hardening suite, and dipole analysis scripts.

The released catalog labels carry a measured spatially-uniform CW-bias residual of 0.26% (9.5σ) attributed

to GZ1 human-handedness training bias propagating through CE-ResNet pseudo-labels and the present ViT-Small classifier. The catalog labels should not be used for precision parity tests below the empirical $\geq 0.75\%$ 50%-rec- 3σ amplitude threshold without local re-normalization of the per-region monopole. Users requiring calibrated soft probabilities for downstream probabilistic models should apply temperature scaling or Platt scaling; the raw p_{eq} values are ranking scores, not frequentist probabilities (see Sec. IV A calibration caveat). The independent GZ1 CW/CCW agreement on the 234,282-galaxy cross-match is 69.91% (Cohen’s $\kappa = 0.40$). The provenance-audit artifacts (`pipelines/p2_chirality/outputs/canonical_provenance/c3_wp_invariance_fsky.json` and `pipelines/p2_chirality/outputs/canonical_provenance/c6_depth_stratified_null.json`) are archived in the repository.

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Facilities: DESI Legacy Imaging Surveys, HuggingFace, RunPod.

Software: Astropy [31], HEALPix/healpy [34, 35], NumPy [36], pandas [37], PyTorch [38], timm [39], NaMaster/pymaster.

AI tool usage: Large language models were used as assistive tools for code review and manuscript preparation. All scientific analyses, results, and conclusions were produced and verified by the author, who takes full responsibility for the content.

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