What Minimal Einstein–Cartan Torsion Does for the Bounce and Cannot Do for Dark Energy

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Popławski's Einstein–Cartan black-hole cosmology replaces the classical singularity with a torsion-supported bounce: eliminating the non-propagating spin connection from the minimal Einstein–Cartan–Holst (ECH) action generates a spin–spin four-fermion contact term that, in the Einstein–Cartan limit $\gamma \to \infty$, reduces to the Hehl–Datta term underlying that bounce mechanism [1–3]. We ask whether the same mechanism can also source late-time dark-energy density and answer, systematically, no. We (i) derive the minimal axial–axial contact interaction $-(3\kappa/16)[\gamma^2/(1+\gamma^2)]J_5^2$, show its direct-channel, hard-cutoff, mean-field NJL scalar projection is repulsive, $G_s = -(3\kappa/16)[\gamma^2/(1+\gamma^2)]$, so the gap equation for this condensate channel has no nonzero solution; (ii) prove a perturbation-transparency theorem: torsion vanishes at every classical perturbation order and the Holst sector decouples identically via the algebraic Bianchi identity; and (iii) catalog fourteen mechanism-class constraints – two derived here, the rest argued or self-labelled heuristics – jointly bounding four channels (NJL contact, one-loop Holst correction, Immirzi running, parity-odd CMB coupling) by which minimal ECH could connect bounce-scale torsion to a late-time Λ-like density, rebutting Popławski's own proposed mechanism. A six-member generating list of dimension-four, rule-admitted local densities (mixed parity, five distinct densities at rank four) is either an exact total derivative on the torsion-free branch, a Fierz-closed $M_{\rm Pl}^{-2}$-suppressed contact term on-shell, or identically vanishing; the on-shell trace-vector irrep is the larger contribution ($\beta/\alpha = 1/(2\gamma) \simeq 2.11$ at benchmark $\gamma = 0.2375$, Ref. [4]), the tensor irrep vanishing identically. The result is a structural dichotomy: the same contact term supplying Popławski's bounce mechanism as $\gamma \to \infty$ is, at finite physical γ, parity-even, Planck-suppressed, and classically transparent to perturbations – unable to generate late-time acceleration. No ECH dark-energy or birefringence prediction is made; this is a channel-level, not operator-level, closure.

I. INTRODUCTION

Minimal Einstein–Cartan–Holst (ECH) gravity promotes the spin connection to an independent field sourced algebraically by fermion spin currents. In loop-quantized cosmology the resulting torsion term replaces the classical singularity with a quantum bounce, and in Popławski's black-hole-cosmology programme the same torsion term is the mechanism by which a collapsing fermionic interior avoids a singularity and instead bounces, producing a new expanding universe inside what an outside observer sees as a black hole [2, 5–7]. The physical content of that mechanism is a spin–spin four-fermion contact interaction: eliminating the non-propagating Cartan connection in favor of the fermionic spin current generates an effective interaction that grows with spin density and becomes repulsive well before the classical curvature singularity is reached, halting collapse. Sec. II derives this contact term explicitly and identifies exactly the sense – Einstein–Cartan limit, repulsive sign – in which it coincides with Popławski's mechanism.

Because the ECH bounce and Popławski's proposed link from bounce-scale torsion to a late-time vacuum energy [3, 5] are built from the same minimal field content, a natural question is whether the contact term responsible for the bounce can *also* source the dark-energy density observed today, $\rho_{\Lambda,\rm obs} \approx (2.25\,\mathrm{meV})^4$. This paper answers that question systematically for the minimal-coupling ECH action. Sec. II derives the contact term from the same Cartan equation that produces Popławski's bounce interaction and states its conditional-sign gap-equation result. Sec. III proves the perturbation-transparency theorem, this paper's sole Tier-I rigorous result. Sec. IV catalogs the fourteen mechanism-class constraints that jointly close the four enumerated dark-energy channels; Sec. V derives the two amplitude-budget route closures; Sec. VI states the operator-list argument with the six densities defined explicitly. Sec. VII draws the positive/negative dichotomy this paper is organized around, and Sec. VIII concludes.

This is a channel-level assessment of the minimal-coupling theory, not an operator-level completeness theorem, and it makes no claim about non-minimal completions, propagating torsion, a dynamical Immirzi field, or quantum loops beyond the stated scope of each result.

II. THE CONTACT TERM: SAME MECHANISM AS THE BOUNCE

We use the first-order ECH action with constant Barbero–Immirzi parameter γ and a minimally coupled

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Dirac field,

$$S[e,\omega,\psi]=\frac{1}{4\kappa}\int\left(\epsilon_{IJKL}+\frac{2}{\gamma}\eta_{I[K}\eta_{L]J}\right)\times e^I\wedge e^J\wedge R^{KL}(\omega)+S_{\rm D}[e,\omega,\psi], \tag{1}$$

$\kappa = 8\pi G = 8\pi/M_{\rm Pl}^2$, mostly-plus signature $\eta_{IJ} = \mathrm{diag}(-,+,+,+)$, $\epsilon_{0123} = +1$. There is no independent torsion-squared term in Eq. (1); any four-fermion expression below is an on-shell result obtained only after the connection equation is solved. Varying ω_{IJ} gives the algebraic equation $\mathcal{Q}_\gamma(e^{[I}\wedge T^{J]}) = \mathcal{J}^{IJ}$ with $\mathcal{Q}_\gamma = \star + \gamma^{-1}\mathbb{1}$ invertible for every real finite nonzero γ [8]. For minimal fermion coupling the spin current is dual to $J_5^I = \bar\psi\gamma^I\gamma^5\psi$, and solving the connection equation and back-substituting (Freidel–Minic–Takeuchi [8], Eqs. 17 and 23, in the normalization bridge $4\pi G = \kappa/2$) gives the on-shell torsion

$$T_{abc}=\alpha\,\varepsilon_{abcd}J^{5d}+\beta\left(\eta_{ab}J^5_c-\eta_{ac}J^5_b\right),\qquad \frac{\beta}{\alpha}=\frac{s_H}{2\gamma}, \tag{2}$$

with $\alpha = \kappa\gamma^2/[2(1+\gamma^2)]$, $\beta = \kappa\gamma/[4(1+\gamma^2)]$, and $s_H = +1$ fixed by the Holst-sign convention of Eq. (1) (the ratio would carry the opposite sign under $s_H = -1$). This is the paper's single normalization, used throughout: every occurrence of α, β, and every four-fermion coefficient below is stated in this normalization, and none is restated in a second, silently different one. Back-substituting Eq. (2) into the torsion-squared term of Eq. (1) gives the minimal axial–axial contact interaction

$$\mathcal{L}_{4\psi}=-\frac{3\kappa}{16}\frac{\gamma^2}{1+\gamma^2}\,(J_5^I J_{5I}). \tag{3}$$

This is the Hehl–Datta four-fermion contact term [1, 9], the Einstein–Cartan generalization of general relativity introduced by Kibble and Sciama [10, 11] and reviewed comprehensively by Shapiro [12]; torsion-sourced cosmological scenarios built on the same contact structure are also studied by Böhmer and Burnett [13]. In the Einstein–Cartan limit $\gamma\to\infty$ (where $\gamma^2/(1+\gamma^2)\to 1$), Eq. (3) reduces exactly to that term, generated by the identical elimination of the non-propagating connection that underlies Popławski's black-hole-cosmology bounce mechanism [2], and it carries the same repulsive sign at every finite γ: writing the contact term's contribution to the effective stress tensor as $\rho_{4\psi} = -\mathcal{L}_{4\psi}$, $p_{4\psi} = \mathcal{L}_{4\psi}$ for a term with no explicit time derivatives, a repulsive (halts collapse) contact interaction requires $\rho_{4\psi} + 3p_{4\psi} < 0$, i.e. $2\mathcal{L}_{4\psi} < 0$, i.e. $\mathcal{L}_{4\psi} < 0$; Eq. (3)'s coefficient $-(3\kappa/16)\gamma^2/(1+\gamma^2) < 0$ multiplying $(J_5^I J_{5I})$ gives $\mathcal{L}_{4\psi} < 0$ whenever $(J_5^I J_{5I}) > 0$, i.e. whenever J^5 is normalized spacelike in the paper's mostly-plus convention – the case realized by a spin-aligned fermion ensemble's axial current in the nonrelativistic, high-spin-density regime this section addresses. No claim is made for a general (e.g. boosted or oscillating) J^5 configuration, nor is a formal signature bridge to Popławski's own conventions asserted beyond this sign-of-pressure check in the regime stated. The Einstein–Cartan coefficient ($\gamma\to\infty$) is the maximal magnitude, since $\gamma^2/(1+\gamma^2) < 1$; at the programme's benchmark $\gamma = 0.2375$ the coefficient is suppressed by $\gamma^2/(1+\gamma^2) = 0.053$ relative to that maximum.

Eq. (2) carries two irreducible pieces: an axial part with coefficient $\alpha\propto\gamma^2/(\gamma^2+1)$ and a trace-vector part with coefficient $\beta\propto\gamma/(\gamma^2+1)$; the tensor irrep vanishes identically for minimal coupling. Because $\alpha\propto\gamma^2/(\gamma^2+1)$ and $\beta\propto\gamma/(\gamma^2+1)$ share the same denominator, the ratio of the two proportionality statements alone gives $\beta/\alpha = 1/\gamma$; the correct ratio $\beta/\alpha = s_H/(2\gamma)$ (Eq. (2)) follows only from the explicit constants $\alpha = \kappa\gamma^2/[2(1+\gamma^2)]$, $\beta = \kappa\gamma/[4(1+\gamma^2)]$ quoted above, whose ratio carries the additional factor of 1/2 that the two bare $\propto$ statements do not display. At the programme's benchmark $\gamma = 0.2375$ this gives $\beta/\alpha = 2.11$: the trace-vector irrep's coefficient is *larger* than the axial one, so the finite-γ on-shell torsion is not well approximated by its purely-axial, Einstein–Cartan reading. That purely-axial reading holds only in the strict limit $\gamma\to\infty$, not at the finite physical γ used elsewhere in this programme (independent on-shell symbolic evaluation [14]).

A. Finite-density benchmark and the NJL sign result

A deliberately elevated homogeneous normalization illustrates only the scale of Eq. (3). With $\kappa n_\psi^2 = (8\pi/M_{\rm Pl}^2)\,n_\psi^2$, $M_{\rm Pl} = 1.22089\times10^{28}\,$eV, and $n_\psi = 100\,\mathrm{cm}^{-3}$, $\kappa n_\psi^2 = 9.954\times10^{-80}\,\mathrm{eV}^4$; dividing by $\rho_{\Lambda,\rm obs} = (2.25\,\mathrm{meV})^4 = 2.563\times10^{-11}\,\mathrm{eV}^4$ gives

$$\frac{\kappa n_\psi^2}{\rho_\Lambda}\simeq 3.884\times10^{-69}\left(\frac{n_\psi}{100\,\mathrm{cm}^{-3}}\right)^2, \tag{4}$$

(3.884×10^{-69} is the value consistent with this paper's single $\rho_{\Lambda,\rm obs} = (2.25\,\mathrm{meV})^4$ used throughout; the alternative 3.6×10^{-69} corresponds instead to $\rho_\Lambda = (2.29\,\mathrm{meV})^4$, an 8%-different normalization of the same observed density, and is not used here), ≈68 orders of magnitude below $\rho_{\Lambda,\rm obs}$ even at an ISM-like number density. This coefficient-one benchmark omits the actual 3/16 and finite-γ factors; number density also does not fix the renormalized composite $\langle J_5^I J_{5I}\rangle$, a vacuum stress tensor, or an equation of state.

The Fierz rearrangement of the axial–axial bilinear $(J_5^I J_{5I})$ closes onto the finite $\{SS, VV, AA, PP\}$ operator basis with the exchange row

$$(J_5^I J_{5I})\;\to\;(\bar\psi\psi)^2+\tfrac12(\bar\psi\gamma^\mu\psi)^2+\tfrac12(\bar\psi\gamma^\mu\gamma^5\psi)^2-(\bar\psi\gamma^5\psi)^2, \tag{5}$$

an independently re-derived result, built from explicit 4×4 Dirac matrices in both metric signatures and an exact Grassmann anticommutation map (rather than a remembered coefficient table) [15]; the scalar-channel coefficient is exactly +1, unique, and signature-independent, with the single Grassmann exchange that maps the ordering (12)(34) → (14)(32) supplying the overall minus sign relative to the c-number Nieves–Pal identity for the same index structure. The declared direct-channel, hard-four-momentum-cutoff, standard mean-field NJL truncation inserts the scalar row of Eq. (5) into the explicitly declared interaction $+G_s(\bar{\psi}\psi)^2$ [16], giving

$$G_s = -\frac{3\kappa}{16}\frac{\gamma^2}{1+\gamma^2} < 0 \tag{6}$$

at every finite γ (Einstein–Cartan limit $\gamma \to \infty$: $G_s \to -3\kappa/16$, the value used for the order-of-magnitude benchmark above), repulsive. In the standard mean-field (Hartree) approximation, the scalar condensate $M \equiv -2G_s\langle\bar{\psi}\psi\rangle$ obeys the self-consistent gap equation

$$M = 2G_s\, M\, I(M, \Lambda_{\rm cut}), \tag{7}$$
$$I(M, \Lambda_{\rm cut}) \propto \int_0^{\Lambda_{\rm cut}} \frac{p^2\, dp}{\sqrt{p^2+M^2}} > 0,$$

with $\Lambda_{\rm cut}$ the declared hard four-momentum cutoff and the proportionality constant a positive color/flavor degeneracy factor $N_c N_f/(4\pi^2)$ from the standard mean-field fermion-loop measure (unmodified from its usual NJL-model form; not re-derived here). Only $I(M, \Lambda_{\rm cut}) > 0$ is load-bearing below, and this holds manifestly (a positive integrand over a positive domain, for any M and any $\Lambda_{\rm cut} > 0$) independent of the prefactor's exact value. The three-line argument: (1) $M = 0$ is always a solution (the symmetric vacuum); (2) any nonzero solution must satisfy $1 = 2G_s\, I(M, \Lambda_{\rm cut})$, but the right-hand side is negative for $G_s < 0$ (Eq. (6)) while the left-hand side is +1, so no nonzero M can satisfy the equation, for any $\Lambda_{\rm cut}$; (3) this is the standard NJL statement that a repulsive scalar-channel coupling does not support dynamical chiral symmetry breaking, so the only solution of Eq. (7) is the trivial one. The real homogeneous scalar gap equation of this convention therefore has no nonzero solution, at finite γ or in the Einstein–Cartan limit alike: this specific truncation admits no condensate. This conditional-sign result does not exclude other truncations, species structures, non-minimal couplings, or propagating torsion; the full regulated treatment with an explicit choice of $\Lambda_{\rm cut}$ and its cutoff-scheme dependence is carried in the archived companion [16].

III. PERTURBATION TRANSPARENCY

The Holst term's vanishing on a torsion-free, matched-data background recovers the standard Holst-action equivalence to Einstein–Cartan gravity for the connection sector [17]; the novel content of the theorem below is that the *same* vanishing extends to every classical perturbative order for canonical scalar matter, at the level of the reduced action rather than only the background field equations – Holst's original result is a statement about the classical background theory, not about perturbations around it.

The catalog's sole Tier-I rigorous result is the following.

Theorem 1 (Perturbation transparency)**.** *Consider the classical ECH action with an invertible tetrad, canonical scalar matter, and real finite nonzero constant γ, on a local patch with matched background, initial, and boundary data (H1: standard falloff; H2: vanishing first-order boundary variation; H3: invertible tetrad; H4: zero fermionic spin density; H5: real finite nonzero γ). Solve the algebraic connection equation before expanding in perturbations. Then the reduced action on the zero-spin branch is the Einstein–scalar action, so the scalar equations and tensor evolution operators coincide with GR at every perturbative order for matched data. Equality of right- and left-helicity* solutions *additionally requires parity-symmetric initial data. Excluded cases: the complex singular values $\gamma = \pm i$ (violates H5), nontrivial global/topological sectors, quantum loops or anomalies, fermion sources (violates H4), non-minimal matter, a dynamical Immirzi field, and propagating torsion are outside this statement.*

Proof. (1) A canonical scalar has zero spin density (H4). (2) The zero source and invertible $\mathcal{Q}_\gamma$ (H3) give $e^{[I} \wedge T^{J]} = 0$; contracting twice with the dual frame shows the kernel is trivial for an invertible tetrad, so $T^I = 0$. (3) The connection reduces to Levi-Civita, $\Gamma^\lambda{}_{\mu\nu} = \mathring{\Gamma}^\lambda{}_{\mu\nu}$. (4) The Holst term $\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}(\mathring{\Gamma})$ vanishes identically on a torsion-free connection by the first (algebraic) Bianchi identity $R_{\mu[\nu\rho\sigma]} = 0$, which holds for any torsionless connection independent of metric compatibility. This Bianchi-vanishing is a single-curvature identity, distinct from the two-curvature Pontryagin density $R\widetilde{R}$; no total-derivative argument is load-bearing. On an FRW background the tensor evolution operators are identical for both helicities, $\mathcal{E}_R = \mathcal{E}_L = \partial_\eta^2 + 2\mathcal{H}\partial_\eta + k^2$, so minimal ECH generates no GW birefringence, tensor chirality, or TB/EB CMB cross-power from parity-symmetric initial data, and the cubic action for ζ receives zero contribution from the Holst term in the stated domain. The theorem is an on-connection-shell equality of local classical reduced actions – not an off-shell equality of the original first-order ECH and Einstein–scalar actions – and its all-orders content follows from the algebraic Cartan constraint and the pointwise Bianchi identity, not an order-by-order calculation. The matched-data hypothesis means: background, initial, and boundary data that are themselves compatible with the zero-spin, torsion-free branch (standard falloff at the boundary, vanishing first-order boundary variation, so no surface term reintroduces torsion-dependent content); data that source torsion on the boundary or at the initial slice are outside the theorem's hypotheses, not a counterexample to it. An extended statement and its own independent second-order verification are also carried in a companion manuscript [16]. □

A. Explicit second-order verification

On the torsion-free branch established above ($T^I = 0$, $\Gamma^\lambda{}_{\mu\nu} = \mathring{\Gamma}^\lambda{}_{\mu\nu}$), expand the Holst dual contraction to second order in metric perturbations. Since it evaluates on the Levi-Civita connection,

$$\mathcal{R}_{\mathrm{H}}(\mathring{\Gamma}) \equiv \tfrac{1}{2}\,\varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}(\mathring{\Gamma}) = 0 \tag{8}$$

identically, by the first algebraic Bianchi identity – at every perturbative order, not only linear order, since the identity is pointwise in the curvature tensor and does not depend on an expansion parameter. The equivalent Nieh–Yan identity $d(e_I \wedge T^I) = T_I \wedge T^I - e_I \wedge e_J \wedge R^{IJ}$ likewise gives zero pointwise when $T^I = 0$; this is not a separate boundary assumption but the same statement in form language. This one-curvature Holst contraction must not be confused with the Pontryagin density $R\widetilde{R}$, which contains two curvature factors and can be nonzero pointwise even on a torsion-free connection – the distinction is what keeps O3 in the topological disposal class of Sec. VI while O1, O6 require the on-shell branch qualifier. Because the Holst contraction vanishes identically rather than merely at leading order, the cubic action for the curvature perturbation ζ – which determines the primordial bispectrum – receives zero contribution from the Holst term in the stated matched-data domain, and the resulting bispectrum is the standard GR one at this order. On an FRW background the tensor mode equation $h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 0$ (primes: conformal-time derivatives, $\mathcal{H} = a'/a$) inherits no parity-dependent modification, giving the identical evolution operators $\mathcal{E}_R = \mathcal{E}_L$ quoted above for both helicities.

B. What would break the transparency

The statement need not survive fermionic spin sources, torsion kinetic terms, non-minimal matter couplings, a dynamical Immirzi field, nonstandard boundary terms, nontrivial global or topological sectors, or quantum loops and anomalies. Each of these changes either the Cartan source, the algebraic character of the connection equation, or the local classical action, and is outside the scope analyzed here; none of them is excluded by assumption elsewhere in this paper, and Sec. IV's B14 entry states explicitly which routes this theorem does and does not bound.

IV. THE BARRIER CATALOG

Each barrier below is tagged with the dark-energy channel(s) it constrains, using the labels defined and closed in detail in Sec. V: **R1** (the NJL contact term of Sec. II), **R2** (one-loop graviton corrections to the Holst term), **R3** (quantum running of the Immirzi parameter), and **R4** (a parity-odd CMB coupling). Seven structural "foundations" (parameter-space, symmetry, and naturalness arguments; Barriers 1–7) and six observational branches (CMB, gravitational-wave, and relic-abundance channels; Barriers 8–14, with Branch H carrying two logically independent entries B8 and B14) together give fourteen distinct mechanism-class constraints that jointly close the four enumerated dark-energy channels (Table I). 'Distinct' and 'mechanism-class' mean no barrier is a logical consequence of another and each probes a separate physical failure mode; this does not assert disjoint assumptions, nor that each is an independently rigorous no-go theorem. Several barriers (B5, B6, B7, B10, B13) are general naturalness or classification arguments rather than sharp ECH-specific calculations, and B9 is an explicitly heuristic closure conditional on stated equilibrium assumptions that realistic quantum bounces can violate; B9 and B14 are never used as stand-alone closures of any route. The sole Tier-I leg among the fourteen is B14 (perturbation transparency, Sec. III), and it is Tier-I strictly on the zero-spin branch its hypotheses cover.

TABLE I. The fourteen catalog entries – fourteen distinct mechanism-class constraints – on minimal-ECH dark-energy routes. B8 and B14 close the same observable channel (tensor parity violation) on disjoint matter branches and are logically independent (Sec. III). *Src.* legend: the seven structural foundations are A (mass-coupling), B (topological shift), C (scalar-tensor universality), D (Planck suppression), E (scale separation), F (attractor sensitivity), G (parameter immunity); the six observational branches are H (parity/tensor-chirality, entered by both B8 and B14), J (Liouville/phase-space), L (UV→IR specificity), L/M (decoupling universality, spanning both), M (vacuum-amplification ceiling), N/O (gravitational democracy). Sequence letters I and K are not used, to avoid confusion with the imaginary unit and the gravitational coupling κ.

#	Barrier	Src.	Blocks
1	Mass-Coupling Lock	A	Torsion as DE
2	Topological-Shift Duality	B	Pseudoscalar mass prot.
3	Scalar-Tensor Universality	C	Distinctive FRW geom.
4	Planck Suppression	D	Disformal coupling
5	Scale Separation	E	Global vacuum coupling
6	Attractor Sensitivity	F	IC transfer to DE
7	Parameter Immunity	G	Cyclic vacuum select.
8	Parity-Even Interaction	H	Tensor chirality (ferm.)
9	Liouville Conservation	J	Reversible selection
10	UV→IR Specificity	L	Generic/specific bridge
11	Decoupling Universality	L/M	Light gauge coupling
12	Vacuum Amplif. Ceiling	M	GW amplitude
13	Gravitational Democracy	N/O	Relics, baryogenesis
14	Perturbation Transparency	H	Tensor chirality (scal.)

B1 – Mass-Coupling Lock (Found. A) [R1]. In Poincaré gauge theory [18, 19], an ultralight torsion mode ($m_T \sim H_0$) requires effective coupling $g_{\mathrm{eff}} \sim 1/(M_{\mathrm{Pl}}\sqrt{|t_3|}) \sim H_0/M_{\mathrm{Pl}} \sim 10^{-61}$, with $\sqrt{|t_3|} \sim m_T^{-1}$ set by the quadratic-torsion-invariant coefficient t_3 of the Hayashi–Shirafuji Lagrangian [18] (a labeled scaling ansatz of the Poincaré-gauge-theory mass spectrum, not a derived equality). Reaching $g_{\mathrm{eff}} \sim 1$ needs $m_T \sim M_{\mathrm{Pl}}$;

keeping $m_T \sim H_0$ against a radiative contribution $\delta m_T^2 \sim M_{\rm Pl}^2$ requires a residual $m_T^2/\delta m_T^2 \sim (H_0/M_{\rm Pl})^2 \sim 10^{-122}$ – the standard cosmological-constant hierarchy.

B2 – Topological-Shift Duality (Found. B) [R1]; in-paper argument, not literature-established. In metric-affine gravity, mass protection $\iff$ no geometric fingerprint: configurations protecting the pseudoscalar mass through topological structure eliminate the geometric content (the field reduces to a standard axion-like particle after torsion elimination), while configurations preserving geometric content cannot protect the mass.

B3 – Scalar-Tensor Universality (Found. C) [R2]. On an FRW background, diffeomorphism invariance forces the torsion fluctuation to couple to the curvature invariants like any other scalar, with no ECH-specific observable [9]: the algebraic torsion equation of motion (Eq. (2)) has no independent kinetic term, so torsion is not a propagating perturbation mode but a background-density-dependent contact coefficient; it tracks $\rho(t)$ identically to the bounce density, decoupling at exactly the same point the contact term itself switches off, yielding no distinctive perturbation signal beyond what the background density already fixes.

B4 – Planck Suppression (Found. D) [R2]. Disformal torsion couplings [12] are Planck-suppressed by $m_\phi^2/M_{\rm Pl}^2$ or $(\partial\phi)^2/M_{\rm Pl}^4$: dimensional counting forces the leading disformal operator $g^{\mu\nu} \to g^{\mu\nu} + f(\phi)\partial^\mu\phi\partial^\nu\phi/M_{\rm Pl}^2$ to carry an explicit $M_{\rm Pl}^{-2}$, so its coefficient scales as $m_\phi^2/M_{\rm Pl}^2$ for a canonically normalized scalar of mass m_ϕ; at cosmological scales ($m_\phi \sim H_0$) these are $\mathcal{O}(10^{-122})$ – observationally inaccessible.

B5 – Scale Separation (Found. E) [R3]; argued in-paper (not literature-established). No mechanism connects the global vacuum integral $\int d^4x\,\sqrt{-g}\,\rho_\Lambda$ to the local bounce density without a means to store and transfer the integrated vacuum energy across $N_{\rm tot} \approx 92$–94 e-folds of inflation; none exists within minimal ECH, whose only algebraic output is the local contact term of Eq. (3).

B6 – Attractor Sensitivity (Found. F) [R3]; argued in-paper (not literature-established). If post-bounce inflation converges to an attractor, bounce initial conditions are washed out before observable scales exit the horizon; if instead the dynamics remain sensitive to those initial conditions, generic inflationary attractor behavior is destabilized. The bounce cannot simultaneously seed a late-time dark-energy value *and* preserve standard inflationary dynamics without a fine-tuned intermediate regime that minimal ECH does not supply.

B7 – Parameter Immunity (Found. G) [R4]; general classification argument. Cyclic-universe vacuum-selection proposals require the Barbero–Immirzi parameter γ to vary across cycles so that different cycles sample different vacua; but γ is a fixed parameter of the theory, not a field with cycle-to-cycle dynamics, set by matching the loop-quantum-gravity black-hole area spectrum [4, 20] (B12, Sec. IV), so minimal ECH provides no landscape of γ values for a selection mechanism to operate on. The scheme-dependence of the numerical window quoted in B12 (entropy-counting convention) is a residual uncertainty in *which* fixed value γ takes, not evidence that γ varies; it supplies no vacuum landscape.

B8 – Parity-Even Interaction (Branch H) [R1]. $(J_5)^2$ is parity-*even* (a Lorentz-scalar product of two axial currents, not a pseudoscalar), so it cannot generate tensor chirality in primordial gravitational waves. B8 is the catalog's constraint on the *fermionic* branch of this observable channel and is logically independent of B14: B14's hypotheses require zero spin density, precisely the condition B8's nonzero J^5 violates, so B14 neither implies nor confirms B8.

B9 – Liouville Conservation (Branch J) [R2]; heuristic. Phase-space volume conservation (Liouville's theorem for a Hamiltonian flow: $\partial_t\rho_{\rm ps} + \{\rho_{\rm ps}, H\} = 0$ preserves phase-space density along any non-dissipative trajectory) prevents irreversible selection among post-bounce states, closing the "vacuum selection at the bounce" escape from Route 2's small one-loop shift: the time-symmetric bounce selects no net dark-energy state. This is a heuristic closure under explicit assumptions (closed non-dissipative evolution, no particle production, no entropy injection); scenarios that break these assumptions evade B9 and are constrained instead by B10–B11, so B9 is never used as a stand-alone closure.

B10 – UV→IR Specificity Dilemma (Branch L) [R3]; argued in-paper (not literature-established). Any bridge from Planck-scale bounce physics to the late-time H_0 scale must be either generic (in which case it explains *any* vacuum energy indiscriminately, undermining its predictive content) or bounce-specific (requiring free parameters tuned to reproduce Λ, which is the tuning problem restated, not solved); no ECH mechanism identified in this paper achieves a non-generic, non-fine-tuned bridge of this kind.

B11 – Decoupling Universality (Branches L/M) [R3]. At low energies all Standard Model gauge fields decouple from the Planck-suppressed torsion sector with the same universal suppression [12, 19], since torsion couples only through the minimal spin connection with no gauge-charge-dependent vertex in the minimal action; minimal ECH cannot preferentially couple to photons, or to whatever degrees of freedom carry the dark-energy density, without introducing new non-minimal, species- or sector-selective couplings outside the scope of this paper.

B12 – Vacuum Amplification Ceiling (Branch M) [R3]. Bounce-epoch gravitational-wave production is bounded above by $\Omega_{\rm GW}^{\rm ECH}|_{\rm bounce} \lesssim (\rho_{\rm crit}/\rho_{\rm Pl})^2 \simeq 0.07$–0.17, using the LQG-bounce critical-density window $\rho_{\rm crit}/\rho_{\rm Pl} \simeq 0.27$–0.41 [16]: the 0.41 endpoint is the canonical effective-LQC value at the standard area-gap choice

$\gamma = 0.2375$ [4], and the 0.27 endpoint substitutes the SU(2) black-hole-entropy value $\gamma \approx 0.274$ [20] into the same $\rho_{\rm crit} = \sqrt{3}/(32\pi^2\gamma^3)\,\rho_{\rm Pl}$ relation – an internal extrapolation across entropy-counting schemes, not a range quoted directly in Ref. [4], so the window is scheme-dependent rather than a single published LQC value; the quadratic scaling is an order-of-magnitude ceiling ansatz, used only as a global bound. This bounce-epoch fraction is not directly comparable to the present-day PTA spectral density $\Omega_{\rm GW}(f_{\rm nHz}) \sim 10^{-9}$ without propagating the bounce spectrum through the transfer function to the nHz band, deferred to a dedicated bounce-GW analysis; B12 closes as a global energy-density-fraction ceiling, not a direct NANOGrav exclusion.

B13 – Gravitational Democracy (Branches N/O) [R4]; structural observation [10, 11]. Torsion couples democratically to all spin-$\frac{1}{2}$ species and cannot preferentially source baryogenesis or relic abundances without species-dependent non-minimal couplings; logically independent of B11 (which bounds the universal gauge-sector magnitude, not species-relative fermion couplings) and of B4 (which fixes the coupling magnitude, not its universality across sectors).

B14 [R2–R4, zero-spin branch, Branch H]. Sole Tier-I leg; Sec. III. Because its hypotheses assume zero spin density, it does not bear on R1 (constrained instead by B1, B2, B8); what it establishes for R2–R4 is that their classical zero-spin baseline is exactly inert, so any signal must come entirely from the quantum or non-minimal ingredient defining each route – bounded by Sec. V.

V. ROUTE 2–4: AMPLITUDE AND NATURALNESS CLOSURES

Route 1 (NJL contact) is closed by Sec. II: amplitude suppressed by $M_{\rm Pl}^{-2}$ and parity-even.

A. Route 2 (one-loop graviton corrections to the Holst term)

Route 2 is anchored in the published one-loop renormalization of the Holst-plus-fermion sector [21]: Shapiro and Teixeira show that the Immirzi parameter runs multiplicatively under one-loop fermion-graviton corrections, with the induced correction to any Holst-sector coupling suppressed by the standard loop factor $\alpha_{\rm em}/(4\pi)$ and by the ratio of the relevant physical scale to $M_{\rm Pl}$ – no anomalous, unsuppressed one-loop enhancement is generated. This paper's ansatz below inherits exactly that loop-suppressed, $M_{\rm Pl}$-suppressed structure. The induced one-loop birefringence shift is bounded, relative to the observed angle $\beta_{\rm obs}$ and the dimensionless strength $M_{\rm Pl}(\alpha/M) \sim 10^{-2}$ of the R4-fitted spectator-ALP coupling (defined below) that reproduces it, by

$$\frac{\Delta\theta_{\text{one-loop}}}{\beta_{\rm obs}\,[M_{\rm Pl}(\alpha/M)]} \sim \frac{\alpha_{\rm em}}{4\pi}\,\frac{H_0/M_{\rm Pl}}{M_{\rm Pl}(\alpha/M)\,\beta_{\rm obs}}. \tag{9}$$

With $\alpha_{\rm em}/(4\pi) \approx 6\times10^{-4}$ rounded up to 10^{-3} (conservative for the suppression claim), $H_0/M_{\rm Pl} \sim 10^{-61}$, $M_{\rm Pl}(\alpha/M) \sim 10^{-2}$ (from the R4-fitted $\alpha/M \sim 10^{-21}\,\text{GeV}^{-1}$ below and $M_{\rm Pl} \sim 10^{19}\,\text{GeV}$), and $\beta_{\rm obs} = 0.342^\circ \approx 6\times10^{-3}\,\text{rad}$, Eq. (9) evaluates to $10^{-3}\cdot 10^{-61}/(10^{-2}\cdot 6\times10^{-3}) \approx 10^{-60}$; allowing two additional orders for unmodeled higher-order corrections gives the quoted suppression of $\gtrsim$58 orders of magnitude in the doubly-normalized ratio of Eq. (9) – normalized by both $\beta_{\rm obs}$ and the fitted coupling strength $M_{\rm Pl}(\alpha/M)$, not by $\beta_{\rm obs}$ alone – so the "58 orders" figure is a statement about this specific ratio, robust to $\mathcal{O}(1)$–$\mathcal{O}(10^{10})$ rescalings of the ansatz-level normalization (leaving $\gtrsim$48 orders even under a ten-order-of-magnitude inflation of the one-loop coefficient). Route 2's *dark-energy* leg is a separate calculation from this birefringence bound: for a constant Nieh–Yan coefficient it is closed at the operator level *modulo the spanning assertion of Sec. VI* (Tier-II, per Table II – not a Tier-I closure) and only at the naturalness level for a dynamical Nieh–Yan coefficient.

B. Route 3 (quantum running of the Immirzi parameter)

A second route is the quantum running of γ itself. The fermion-coupled one-loop β-function of Benedetti–Speziale [22] is

$$\mu\,\frac{\partial\gamma^2}{\partial\mu} = -(\gamma^2-1)\,\frac{\mu^2\tilde\kappa^2}{(8\pi)^2}\,(23\gamma^2+5), \qquad \tilde\kappa^2 \equiv 16\pi G, \tag{10}$$

whose only fixed point is the ultraviolet-attractive $\gamma^2 = 1$ (formally outside perturbative control). The explicit $\mu^2\tilde\kappa^2 \simeq (\mu/M_{\rm Pl})^2$ prefactor makes the running power-suppressed, not logarithmic, so $|\Delta\gamma/\gamma| \sim (\mu_{\rm UV}/M_{\rm Pl})^2$, dominated by the ultraviolet endpoint. Throughout this integration $G = 1/M_{\rm Pl}^2$ with $M_{\rm Pl} = 1.22\times10^{19}\,\text{GeV}$ (the non-reduced Planck mass, as used consistently elsewhere in this paper, e.g. Sec. II). Integrating Eq. (10) from a GUT-scale boundary $\mu_{\rm UV} \sim 10^{16}\,\text{GeV}$ down to $\mu_{\rm IR} \sim 1\,\text{GeV}$ at $\gamma \approx 0.24$ gives $|\Delta\gamma/\gamma| \approx 1.4\times10^{-6}$. Propagated to the dark-energy channel using the explicit Tier-III (order-of-magnitude ansatz, not a rigorous derivation) scaling relation

$$\frac{\delta\rho_\Lambda^{(\rm R3)}}{\rho_{\Lambda,\rm obs}} \sim \left|\frac{\Delta\gamma}{\gamma}\right| \times \frac{\kappa n_\psi^2}{\rho_{\Lambda,\rm obs}}, \tag{11}$$

which propagates the fractional Immirzi-parameter shift linearly onto the already-established Route 1 contact-term magnitude $\kappa n_\psi^2 \simeq 9.954\times10^{-80}\,\text{eV}^4$ at the benchmark fermion density $n_\psi \sim 100\,\text{cm}^{-3}$ (Sec. II), for which $\kappa n_\psi^2/\rho_{\Lambda,\rm obs} \simeq 3.9\times10^{-69}$. With $|\Delta\gamma/\gamma| \approx 1.4\times10^{-6}$ this gives $\delta\rho_\Lambda^{(\mathrm{R3})}/\rho_{\Lambda,\mathrm{obs}} \sim 5.5\times10^{-75}$, i.e. suppression by ≈ 74 **orders of magnitude** below $\rho_{\Lambda,\mathrm{obs}}$; varying the sub-Planckian UV boundary $\mu_{\mathrm{UV}} \in [10^{16}\,\mathrm{GeV}, M_{\mathrm{Pl}}]$ across its theoretically allowed range (through the power-law $|\Delta\gamma/\gamma| \propto (\mu_{\mathrm{UV}}/M_{\mathrm{Pl}})^2$ of Eq. (10)) moves this order-of-magnitude estimate across a 68–74-order-of-magnitude window below $\rho_{\Lambda,\mathrm{obs}}$; only a literal Planck-scale UV boundary would push $|\Delta\gamma/\gamma| \to \mathcal{O}(1)$, the regime the β-function itself flags as outside perturbative control, so the closure holds for any sub-Planckian UV boundary across the full quoted window. (A distinct, chiral-fermion-counting relation used in the companion no-go survey, $(\Delta\gamma/\gamma)(H_0/M_{\mathrm{Pl}})$ with $H_0/M_{\mathrm{Pl}} \simeq 1.18 \times 10^{-61}$ [23], gives a different 61–67-order window at $\Delta\gamma/\gamma \in [1.4 \times 10^{-6}, 0.3]$; the two windows differ because they propagate the Immirzi-parameter shift through two different physical relations – this section's Eq. (11) amplitude ratio versus the survey's dimensional H_0/M_{Pl} ratio – not because either is in error.)

C. Route 4 (parity-odd CMB coupling)

Route 4 (a spectator axion-like particle, or the fermion axial current directly, coupled to $R\widetilde{R}$) is not amplitude-suppressed: a free-normalization fit reproduces both the WMAP+Planck birefringence signal $\beta_{\mathrm{obs}} = 0.342^\circ \pm 0.094^\circ$ [24, 25] (comparable to ACT DR6, $\beta = 0.215^\circ \pm 0.074^\circ$ [26]) *and* ρ_Λ, giving the anchor value $\alpha/M \sim 10^{-21}\,\mathrm{GeV}^{-1}$ used in Eq. (9) above. Both birefringence measurements are statistical indications rather than established detections – $\approx 3.6\sigma$ for WMAP+Planck and $\approx 2.9\sigma$ for ACT DR6, significances obtained from different datasets and distinct null procedures and therefore not directly comparable as statistical weights – and the Route-2 suppression conclusion above is insensitive to that status: a smaller or vanishing true birefringence signal only widens the margin of Eq. (9). Minimal ECH derives neither the required ultralight mass $m_\theta \sim H_0$ nor the fitted coupling α/M from first principles – it relocates rather than solves the cosmological-constant problem, so R4 closes as a naturalness/explanatory-deficit objection, not an amplitude exclusion.

The R1–R3 legs achieve mathematical amplitude closure (suppressed by explicit M_{Pl} powers and loop factors, many orders below observable thresholds regardless of normalization); R4 achieves naturalness/explanatory closure. These are logically distinct closure modes, classified separately above. The phenomenological parameter α/M that fits β_{obs} is therefore best understood as an R4-bounded coupling with the dark-energy density supplied by an unrelated sector, rather than by ECH-internal physics.

D. Engaging Popławski's own dark-energy proposal

Popławski himself proposes a torsion-sourced cosmological constant, identifying the same spin–spin contact interaction that supports his bounce with a present-day vacuum energy density set by the average cosmic fermion (baryon) spin density [5]. That proposal maps directly onto **Route 1** (NJL contact) in this paper's R1–R4 taxonomy: it is the same $-(3\kappa/16)[\gamma^2/(1+\gamma^2)]J_5^2$ interaction of Eq. (3), evaluated at the cosmic fermion number density n_ψ rather than at the collapsing-star density that sources the bounce. Sec. II already supplies the closing argument at the same order-of-magnitude benchmark used there ($n_\psi \sim 100\,\mathrm{cm}^{-3}$, ISM-like): $\kappa n_\psi^2 \simeq 9.954 \times 10^{-80}\,\mathrm{eV}^4$, which is $\kappa n_\psi^2/\rho_{\Lambda,\mathrm{obs}} \simeq 3.9 \times 10^{-69}$ – sixty-eight orders of magnitude too small to source the observed density at this benchmark, and the repulsive sign of G_s (Eq. (6)) means the mean-field gap equation (Eq. (7)) additionally admits no nonzero condensate at any finite γ, so no larger effective density is generated by symmetry breaking. The identification is therefore R1, and R1 is closed by both the amplitude argument (order 10^{-69} at benchmark density, and the actual present-day cosmic baryon density $n_b \sim 0.25\,\mathrm{m}^{-3}$ is smaller still, widening rather than narrowing this gap) and the sign argument (Eq. (6)–(7)). This paper's negative result is accordingly not merely a generic no-go over an enumerated channel list but a direct, quantitative rebuttal of Popławski's own stated dark-energy mechanism, evaluated in his own proposed channel at his own order of magnitude.

VI. THE OPERATOR LIST: RANK, NOT BASIS

The barrier catalog and route closures above bound the amplitude of representative parity-odd operators. A referee may reasonably ask whether the conclusion survives the full local operator space. Under the construction rule of zero additional derivative order, ε-contraction (mixed parity, not strictly parity-odd), full index contraction to a scalar density, and mass dimension exactly four, the admitted densities built from the tetrad e^I_μ, the curvature two-form of the torsionful connection, the algebraically-fixed torsion of Eq. (2), and the minimal axial current J^5 are spanned by the six-member list

$$\begin{aligned} \mathcal{O}_1^{[4]} &= M_{\mathrm{Pl}}^2\,\varepsilon^{\mu\nu\rho\sigma} e^I_\mu e^J_\nu R_{IJ\rho\sigma}, & \mathcal{O}_2^{[4]} &= M_{\mathrm{Pl}}^2\,\mathrm{NY}, \\ \mathcal{O}_3^{[4]} &= R^{IJ} \wedge R_{IJ}, & \mathcal{O}_4^{[4]} &= M_{\mathrm{Pl}}^2\,\varepsilon^{\mu\nu\rho\sigma} T^I{}_{\mu\nu} T_{I\rho\sigma}, \\ \mathcal{O}_5^{[4]} &= \varepsilon^{\mu\nu\rho\sigma} T^I{}_{\mu\nu} e_{I\rho} J^5_\sigma, & \mathcal{O}_6^{[4]} &= M_{\mathrm{Pl}}^2\,\varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}, \end{aligned} \tag{12}$$

where $\mathrm{NY} \equiv \partial_\mu(\varepsilon^{\mu\nu\rho\sigma} e_{I\nu} T^I{}_{\rho\sigma})$ is the Nieh–Yan density and R is the curvature of the full torsionful connection; the explicit M_{Pl}^2 prefactors on four of the six entries promote densities that would otherwise sit at naive dimension $+2$ to genuine dimension 4 (O3 and O5 already sit

TABLE II. Evidentiary status of each leg: (I) rigorous within stated scope, (II) structural argument, (III) ansatz-level dimensional estimate. No leg is claimed more strongly than recorded here.

Leg	Status
Perturbation transparency	**(I)** Rigorous within stated scope.
R1 (NJL contact)	**(II)+(III)**: parity-even structure is algebraic; $M_{\rm Pl}^{-2}$ suppression is an ansatz-level estimate.
R2 (one-loop)	**(III)** birefringence leg; **(II)** dark-energy leg (both branches).
R3 (Immirzi running)	**(II)+(III)**: power-law suppression $\lvert\Delta\gamma/\gamma\rvert \propto (\mu_{\rm UV}/M_{\rm Pl})^2$ is structural; the scaling relation of Eq. (11) is a Tier-III order-of-magnitude ceiling.
R4 (CMB/spectator-ALP)	**(II)** structural naturalness objection, not an amplitude exclusion.

at dimension 4 with a bare dimensionless coefficient). Independent symbolic adjudication [27], expanding all six over a common basis of independent jet monomials in exact rational arithmetic, establishes that this list has **rank four** as a set of densities, with a two-dimensional null space

$$\mathcal{O}_1^{[4]} - \mathcal{O}_6^{[4]} = 0, \qquad 2\,\mathcal{O}_1^{[4]} + 2\,\mathcal{O}_2^{[4]} - \mathcal{O}_4^{[4]} = 0, \tag{13}$$

the second equivalently $\mathcal{O}_1^{[4]} = \frac{1}{2}\mathcal{O}_4^{[4]} - \mathcal{O}_2^{[4]}$; both relations hold identically off shell and on shell. The first relation is not an independent computational result: because the connection varied throughout is a metric-compatible $so(1,3)$ spin connection, the tetrad converts frame to coordinate indices exactly, so $\varepsilon^{\mu\nu\rho\sigma} e^I_\mu e^J_\nu R_{IJ\rho\sigma} \equiv \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$ identically – $\mathcal{O}_1^{[4]}$ and $\mathcal{O}_6^{[4]}$ as defined in Eq. (12) are the same density written twice. The six-member list therefore contains **five distinct densities**; its rank is four, with a two-dimensional null space of which one direction is exactly this tetrad-conversion duplication and only the second relation, $2\mathcal{O}_1^{[4]} + 2\mathcal{O}_2^{[4]} - \mathcal{O}_4^{[4]} = 0$, carries independent content. **Rank two** modulo total derivatives follows because O2 and O3 are themselves total derivatives on any connection. This is a deliberately redundant generating set of recognizable invariants, *not* a linearly independent basis; the list is asserted to span the rule-admitted space from the construction rule itself, not by exhaustive enumeration. The duplication is retained (rather than dropping O6) because both entries are independently recognizable invariants in the literature.

Every member reduces to one of three classes, evaluated on the on-shell ECH branch of Eq. (2): an exact total derivative (O2, the Nieh–Yan density; O3, the Pontryagin density), identically vanishing on the *torsion-free* branch only (O1, O6: the single-curvature parity-odd densities vanish identically for any Riemann tensor obeying the algebraic Bianchi identity $R_{\mu[\nu\rho\sigma]} = 0$, but the on-shell ECH torsion is nonzero, so that identity does not apply on shell), or a Fierz-closed four-fermion contact term suppressed by $M_{\rm Pl}^{-2}$ (O4, O5, and the O1=O6 remainder). Concretely: (a) *O1, O6, on shell.* The torsion-free first Bianchi identity used to kill O1 and O6 is violated by a nonzero on-shell torsion; on the ECH branch Eq. (13) instead gives $\mathcal{O}_1^{[4]} = \mathcal{O}_6^{[4]} = -\mathcal{O}_2^{[4]} + \frac{1}{2}\mathcal{O}_4^{[4]}$ exactly – minus the Nieh–Yan total derivative (zero equation-of-motion and vacuum-energy content) plus half the Fierz-closed $\mathcal{O}_4^{[4]}$ below. O1 and O6 are therefore *not* exact total derivatives on shell; their remainder sits in the same disposal class as O5. (b) *O5.* Using $S^{abc} = \frac{1}{4}\varepsilon^{abcd} J^5_d$, O5 reduces under Eq. (2) to the single-normalization value

$$\mathcal{O}_5^{[4]} = -3\kappa\,\frac{\gamma^2}{1+\gamma^2}\,(J^5\!\cdot\! J^5), \tag{14}$$

which the Fierz-by-Fierz projection lemma closes onto the finite $\{SS, VV, AA, PP\}$ basis with no escape operator and no change of $M_{\rm Pl}^{-2}$ power. This density is parity-*even* both off shell and on shell – a Lorentz-scalar product of two axial currents is P-even (B8); O5 is admitted into the list by the *ε-construction rule* (zero extra derivative order, full contraction to a scalar density, mass dimension four), not by being P-odd, so the six-member list is a mixed-parity ε-contracted set, not a strictly parity-odd one. Only the axial irrep of Eq. (2) contributes, the trace-vector part dropping out of O5 entirely. (c) *O4.* $T_I \wedge T^I$ requires *both* the axial and the trace-vector torsion irreps present simultaneously – it vanishes on a pure axial or a pure trace-vector torsion alike, and is carried entirely by the axial×trace-vector cross term – and the on-shell torsion of Eq. (2) populates exactly that channel:

$$\begin{aligned}\mathcal{O}_4^{[4]} &= -24\,M_{\rm Pl}^2\,\alpha\beta\,(J^5\!\cdot\! J^5) = -\frac{24\pi\kappa\gamma^3}{(1+\gamma^2)^2}\,(J^5\!\cdot\! J^5)\\ &= -\frac{192\pi^2 G\gamma^3}{(1+\gamma^2)^2}\,(J^5\!\cdot\! J^5),\end{aligned} \tag{15}$$

using $\kappa = 8\pi G = 8\pi/M_{\rm Pl}^2$ as defined above. The same Fierz-closed $(J^5\cdot J^5)$ structure as O5 at the same $M_{\rm Pl}^{-2}$ power, with $\mathcal{O}_4^{[4]}/\mathcal{O}_5^{[4]} = 8\pi\gamma/(1+\gamma^2) \simeq 5.65$ at $\gamma = 0.2375$ – O4 is the larger of the two by a factor of ~ 5.7; it vanishes in the $\gamma \to \infty$ Einstein–Cartan limit, where the trace-vector irrep switches off, and, degenerately, as $\gamma \to 0$, so the qualitative no-go conclusion (O4 and O5 are the same $M_{\rm Pl}^{-2}$-suppressed contact structure) is unaffected by the size correction. (d) *The ε-free companion.* The same on-shell torsion also renders the torsion–current density, which carries no ε tensor and so is not itself in the construction rule's {O1–O6} span,

$$T^{a}{}_{ab}\,J^{5b} = 3\beta\,(J^5\!\cdot\!J^5), \qquad \beta = \frac{\kappa\gamma}{4(1+\gamma^2)}, \tag{16}$$

nonzero (the axial irrep drops out of the trace; only the trace-vector irrep survives). Before torsion elimination this density is parity-odd – a polar torsion trace vector contracted with the axial current – and it is precisely the trace-vector content that is *not* in the excluded (non-minimal-only) set: it is generated by the Holst term under strictly minimal coupling and is carried throughout, entering the operator list through $\mathcal{O}_4^{[4]}$, through the O1 = O6 remainder, and through Eq. (16) itself. Its value is the same Fierz-closed $(J^5\cdot J^5)$ structure at the same $M_{\rm Pl}^{-2}$ power as O4 and O5, so it joins that bounded class and enlarges neither the closure argument's scope nor its conclusion. Hence every admissible local dimension-4 ε-contracted (construction-rule-admitted, mixed-parity) density in minimal ECH is topological, Fierz-basis-reducible, or (on the torsion-free branch only) identically vanishing – and the Fierz-basis-reducible class, carrying O4, O5, and the O1=O6 remainder, is bounded uniformly at $M_{\rm Pl}^{-2}$, so none produces a $({\rm meV})^4$ vacuum energy without a new light scale $\mu \ll M_{\rm Pl}$ or a protected symmetry – either of which is itself the tuning to be explained. This does not change the closure stated in Sec. V: O4's and the O1=O6 remainder's surviving content is still $M_{\rm Pl}^{-2}$-suppressed and inside the same Fierz-closed basis as O5, so Route 2's dark-energy leg for constant Nieh–Yan coefficient is unaffected, only its stated mechanism is corrected relative to a purely-axial ($\gamma \to \infty$) reading of the on-shell torsion.

VII. DISCUSSION

The two halves of this paper's title are the same calculation read in two directions. Eliminating the non-propagating ECH connection in favor of the fermionic spin current generates exactly one interaction – Eq. (3) – and that interaction is simultaneously (a) repulsive at high spin density at every finite γ, and identical in the $\gamma \to \infty$ reduction to the Hehl–Datta term that is the physical content of Popławski's torsion-avoided singularity and bounce mechanism, and (b) parity-even, suppressed at the programme's finite γ by $\gamma^2/(1+\gamma^2) = 0.053$ relative to that Einstein–Cartan magnitude, Planck-suppressed, and (on the zero-spin branch) classically transparent to every scalar and tensor perturbation – exactly the properties a viable dark-energy source cannot have. Minimal ECH is not a two-purpose mechanism with one purpose realized and the other merely unproven; the barrier catalog of Sec. IV shows that every distinct route from the bounce-scale torsion sector to a late-time Λ-like density is closed by a separate, named failure mode, and Sec. VI shows those failure modes are not an artifact of an incomplete operator enumeration.

This closure is specific to the ECH mechanism class under minimal coupling; it says nothing about non-minimal torsion couplings, propagating torsion, a dynamical Immirzi field, or the bounce's predicted galaxy-spin observable, which is tested independently by the DESI-scale chirality survey [28]. Nor does it bear on bounce cosmology generally: the matter-bounce $f_{\rm NL} = -35/16$ signature and NANOGrav-band gravitational-wave forecasts are properties of the matter-bounce class, not of ECH torsion, and are pursued in a separate research line [29]. What this paper closes is narrower and more useful to state cleanly: the specific claim that the same minimal spin-torsion sector producing the ECH bounce also supplies the observed dark-energy density.

A. What is established and what is not

Fourteen distinct mechanism-class constraints, covering seven foundational classes and six observational branches, collectively close the four enumerated minimal-ECH dark-energy channels (R1–R4) under stated assumptions. Two of those channels (R2, R3) are closed at the amplitude-budget level – Route 2 with ≈60 (conservatively ≥ 58) orders of margin in the doubly-normalized ratio of Eq. (9), Route 3 with ≈ 74 (68–74) orders against the observed dark-energy density – and both amplitude closures are robust to $\mathcal{O}(1)$–$\mathcal{O}(10^{10})$ rescalings of their ansatz-level normalizations. The operator-list argument (Sec. VI) extends the governing dimensional bound from one representative operator to the exhibited six-member, five-distinct-density generating list {O1–O6} (mixed parity, rank four), with no escape operator identified within its span; that the list *spans* the rule-admitted space is asserted from the stated construction rule, not proved by exhaustive symbolic enumeration.

What is *not* established is a fully operator-level no-go over the entire space of diffeomorphism-invariant gravitational operators: the argument is scoped to local, gauge-invariant, mass-dimension-exactly-four densities built from the minimal-ECH field content. It does not enumerate derivative four-fermion terms suppressed by additional momentum powers, curvature-torsion mixed invariants of higher order, multi-species chiral structures beyond the single-species minimal content, dynamical-Immirzi-field completions, or the tensor (16) torsion irrep and the other torsion couplings that only a relaxation of minimal coupling admits – any of these is a genuine escape from the minimal-coupling scope, not a loophole within it. The trace-vector (4) irrep is explicitly *not* in that excluded set (Sec. VI): it is generated by the Holst term under strictly minimal coupling and is carried throughout this paper's closure. A non-minimal completion introducing a new light scale $\mu \ll M_{\rm Pl}$, or a protected symmetry zeroing the Planck-scale piece, would itself evade the single-scale bound – but such a scale or symmetry is itself the fine-tuning the mechanism was meant to explain, so evading the no-go this way relocates rather than solves the cosmological-constant problem.

The barrier catalog is a map of failure modes of mixed evidentiary strength, not fourteen independently decisive theorems: five entries (B5, B6, B7, B10, B13) are general naturalness or classification arguments rather than ECH-specific calculations, and one (B9) is an explicitly heuristic closure conditional on stated equilibrium assumptions that dissipative or particle-producing bounces can violate. Only the perturbation-transparency theorem (B14, Sec. III) enters the closure table as a Tier-I result, and its route reach is fixed by its zero-spin-density hypotheses, not asserted beyond them.

VIII. CONCLUSIONS

Minimal Einstein–Cartan–Holst torsion, under strictly minimal fermion coupling, generates one physical interaction: a spin–spin four-fermion contact term, repulsive at high density at every finite γ, and identical in the Einstein–Cartan limit $\gamma \to \infty$ to the Hehl–Datta term underlying Popławski's torsion-avoided singularity and black-hole-cosmology bounce. This paper shows, at channel-amplitude granularity and within a stated minimal-coupling scope, that the same interaction cannot additionally source the observed late-time dark-energy density: its NJL scalar projection admits no condensate in the declared truncation at any finite γ (Sec. II), it decouples identically from classical perturbations on the zero-spin branch by a rigorous Bianchi-identity theorem (Sec. III), fourteen mechanism-class constraints jointly close all four enumerated dark-energy channels with explicit amplitude budgets (Sec. IV–V), and a rank-four generating list of the explicitly defined dimension-four, construction-rule-admitted (mixed-parity) operators, spanning five distinct densities, including the on-shell trace-vector content the purely-axial reading omits, shows this conclusion is not an artifact of an incomplete operator basis (Sec. VI). The positive result – what minimal ECH does for the bounce – and the negative result – what it cannot do for dark energy – are two readings of the same algebraic elimination, and this paper's contribution is to state both precisely, with the closure scope and evidentiary strength of every claim recorded explicitly rather than implied.

Data and Code Availability

The symbolic verification scripts supporting the transparency theorem, the on-shell torsion-irrep computation, and the operator-list adjudication are publicly available in the repository `https://github.com/Hubify-Projects/bigbounce`, pinned to commit `ded46bc5df8d39bbaac7bfbee16b07f0376bab34`:

- `dim4_parityodd_enumeration.py` – verifies the two reduction identities and the two exact rank-four relations of Eq. (13).
- `operator_basis_adjudication_2026_08_07.py` (report: `...adjudication_2026_08_07.md`) – re-derives O1–O6 from the Cartan structure equations on an algebraically independent 2-jet and certifies the rank-four/null-space result.
- `ech_torsion_onshell_2026_08_08.py` (report: `...torsion_onshell_2026_08_08.md`) – solves Eq. (2) on six explicitly curved on-shell ECH configurations and verifies $\mathcal{O}_1^{[4]} = \mathcal{O}_6^{[4]}$ and Eq. (16) on shell.
- `fierz_adjudication_2026_08_05.py` (report: `...fierz_adjudication_2026_08_05.md`) – builds explicit 4×4 Dirac matrices in both signatures, solves the full 5×5 Fierz matrix, and certifies the scalar-channel coefficient of Eq. (6).

The full regulated NJL derivation and its own verification scripts are carried in a companion manuscript, archived as an immutable deposit at doi:10.5281/zenodo.21481838 (CC-BY-4.0) [16]; a companion systematic no-go survey carrying the barrier-catalog and route derivations in full detail is `paper1c_nogo_survey/main.tex`, pinned to the same commit [23].

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[1] F. W. Hehl and B. K. Datta, J. Math. Phys. **12**, 1334 (1971).

[2] N. J. Popławski, Physics Letters B **694**, 181 (2010), arXiv:1007.0587 [astro-ph.CO].

[3] N. J. Popławski, General Relativity and Gravitation **44**, 491 (2012), arXiv:1102.5667 [gr-qc].

[4] A. Ashtekar and P. Singh, Classical and Quantum Gravity **28**, 213001 (2011), arXiv:1108.0893 [gr-qc].

[5] N. J. Popławski, Annalen der Physik **523**, 291 (2011), arXiv:1005.0893 [gr-qc].

[6] N. J. Popławski, The Astrophysical Journal **832**, 96 (2016), arXiv:1410.3881 [gr-qc].

[7] M. P. Unger and N. J. Popławski, The Astrophysical Journal **870**, 78 (2019), arXiv:1808.08327 [gr-qc].

[8] L. Freidel, D. Minic, and T. Takeuchi, Physical Review D **72**, 104002 (2005), arXiv:hep-th/0507253 [hep-th].

[9] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, Reviews of Modern Physics **48**, 393 (1976).

[10] T. W. B. Kibble, Journal of Mathematical Physics **2**, 212 (1961).

[11] D. W. Sciama, Reviews of Modern Physics **36**, 463 (1964).

[12] I. L. Shapiro, Physics Reports **357**, 113 (2002), arXiv:hep-th/0103093.

[13] C. G. Böhmer and J. Burnett, Physical Review D **78**, 104001 (2008), arXiv:0809.0469.

[14] H. Golden, On-shell torsion irrep content of minimal Einstein–Cartan–Holst gravity at finite Barbero–Immirzi parameter, `https://github.com/Hubify-Projects/bigbounce/blob/ded46bc5df8d39bbaac7bfbee16b07f0376bab34/research/theory_audit/ech_torsion_onshell_2026_08_08.md` (2026), Independent symbolic adjudication: on-shell ECH torsion carries axial and trace-vector irreps at finite γ (ratio $\beta/\alpha = 1/2\gamma$), tensor irrep absent; corrects an earlier purely-axial reading valid only in the Einstein–Cartan limit $\gamma \to \infty$.

[15] H. Golden, Fierz rearrangement of the minimal Einstein–Cartan–Holst axial–axial contact term: independent adjudication, `https://github.com/Hubify-Projects/bigbounce/blob/ded46bc5df8d39bbaac7bfbee16b07f0376bab34/research/theory_audit/fierz_adjudication_2026_08_05.md` (2026), Independent symbolic adjudication built from explicit 4×4 Dirac matrices in both signatures and an exact Grassmann anticommutation map: the scalar-channel Fierz coefficient of $(J_5^I J_{5I})$ is exactly $+1$, unique and signature-independent.

[16] H. Golden, Zenodo 10.5281/zenodo.21481838 (2026), Companion paper (Paper I A), publicly archived on Zenodo under version DOI 10.5281/zenodo.21481838 (concept DOI 10.5281/zenodo.21481837), deposited 2026 July 22 under CC-BY-4.0. Public permanent archive of the manuscript and its exact source; not an arXiv preprint and not peer reviewed.

[17] S. Holst, Phys. Rev. D **53**, 5966 (1996).

[18] K. Hayashi and T. Shirafuji, Phys. Rev. D **19**, 3524 (1979).

[19] M. Blagojević and F. W. Hehl, *Gauge Theories of Gravitation: A Reader with Commentaries* (Imperial College Press, London, 2013).

[20] A. Ghosh and P. Mitra, Phys. Lett. B **616**, 114 (2005), arXiv:gr-qc/0411035 [gr-qc].

[21] I. L. Shapiro and P. M. Teixeira, Classical and Quantum Gravity **31**, 185002 (2014), arXiv:1402.4854 [gr-qc].

[22] D. Benedetti and S. Speziale, J. Phys. Conf. Ser. **360**, 012011 (2012), arXiv:1111.0884 [hep-th].

[23] H. Golden, Structural No-Go Survey of Minimal Einstein–Cartan–Holst Dark-Energy Routes: Barrier Catalog and Operator-List Argument, `https://github.com/Hubify-Projects/bigbounce/tree/ded46bc5df8d39bbaac7bfbee16b07f0376bab34/arxiv/paper1c_nogo_survey` (2026), Companion no-go survey (Paper I C) carrying the full barrier-catalog and route derivations in detail; a repository draft, not independently submitted to arXiv.

[24] J. R. Eskilt and E. Komatsu, Phys. Rev. D **106**, 063503 (2022), arXiv:2205.13962 [astro-ph.CO].

[25] Y. Minami and E. Komatsu, Physical Review Letters **125**, 221301 (2020), arXiv:2011.11254 [astro-ph.CO].

[26] P. Diego-Palazuelos and E. Komatsu, arXiv preprint (2025), arXiv:2509.13654 [astro-ph.CO].

[27] H. Golden, Operator-basis independence adjudication for the minimal-ECH dimension-four parity-odd operator list, `https://github.com/Hubify-Projects/bigbounce/blob/ded46bc5df8d39bbaac7bfbee16b07f0376bab34/research/theory_audit/operator_basis_adjudication_2026_08_07.md` (2026), Independent symbolic computation: the six-member list {O1–O6} has rank four as a set of densities and rank two modulo total derivatives – a generating set, not a linearly independent basis.

[28] H. Golden, Zenodo 10.5281/zenodo.21461899 (2026), Companion paper (Paper IV), publicly archived on Zenodo under version DOI 10.5281/zenodo.21461899 (concept DOI 10.5281/zenodo.21461898), deposited 2026 July 20 under CC-BY-4.0. Public permanent archive of the manuscript and its exact source; not an arXiv preprint and not peer reviewed.

[29] H. Golden, Zenodo 10.5281/zenodo.21461881 (2026), Companion paper (Paper II), publicly archived on Zenodo under version DOI 10.5281/zenodo.21461881 (concept DOI 10.5281/zenodo.21461880), deposited 2026 July 20 under CC-BY-4.0. Public permanent archive of the manuscript and its exact source; not an arXiv preprint and not peer reviewed.

[30] S. Mercuri, Physical Review Letters **103**, 081302 (2009), arXiv:0902.2764 [gr-qc].