A Structural No-Go Survey of Minimal Spin-Torsion Routes to Dark Energy and Bounce Phenomenology

Houston Golden[1, *]
[1] *Independent Researcher, Los Angeles, California, USA*
(Dated: August 7, 2026 (v1C.0.12))

We present a structural no-go survey of the minimal Einstein–Cartan–Holst (ECH) spin-torsion framework as a source of late-time dark energy and bounce phenomenology. Working from the algebraic torsion-elimination setup established in a companion paper, we catalog thirteen distinct mechanism-class constraints — fourteen historical catalog entries, one subsumed by another — spanning seven foundational mechanism classes and six observational-channel branches, each closing one or more of the four routes by which the ECH bounce could plausibly source a Λ-like late-time density. Of the four parity-odd or dark-energy channels enumerated by this framework, we present amplitude-budget closures of the two historical quantum routes: Route 2 (one-loop graviton-sector corrections to the Holst term), whose budget is anchored in the published one-loop renormalization of the Holst-plus-fermion sector and closes against the *observed birefringence amplitude* with roughly sixty orders of magnitude (conservatively ≥ 58) of suppression margin, and Route 3 (quantum running of the Barbero–Immirzi parameter), bounded by an integrated one-loop renormalization-group flow that leaves the contribution 61–67 orders of magnitude below the *observed dark-energy density* (the $\sim$67-order endpoint from the derived integrated flow, the $\sim$61-order endpoint from a deliberately pessimistic chiral-count bound). Neither route is presented as a viable mapping: the companion paper does not retain these historical mappings as results, and this survey closes them as bounded amplitude budgets. We further present an operator-list argument: a six-member *spanning list* {O1–O6} of local, gauge-invariant, diffeomorphism-covariant parity-odd densities of mass dimension exactly four, exhibited under stated construction rules for the minimal-coupling ECH field content, is shown to close — every member is a topological total derivative, a Fierz-closed four-fermion contact term suppressed by $M_{\rm Pl}^{-2}$, or identically vanishing (the single-curvature pair by the algebraic Bianchi identity on the torsion-free branch, where on the $T = \kappa S$ branch it instead reduces to an exact total derivative; the ε-contracted torsion-square under the purely axial Cartan torsion). The list is deliberately redundant: independent symbolic computation gives it rank four (two exact relations, O1 = O6 and O1 = $\frac{1}{2}$O4 − O2), and rank two modulo total derivatives, so it is a generating set of recognizable invariants and not a linearly independent basis. That the list *spans* the rule-admitted operator space is asserted from the construction rules, not proved by exhaustive symbolic enumeration. The catalog carries an explicit evidentiary classification (rigorous result / structural argument / ansatz-level estimate): only the perturbation-transparency result is a Tier-I rigorous theorem, and the survey is a channel-level, not operator-level, closure within a stated minimal-coupling scope — non-minimal completions, an added light scale or non-minimal torsion coupling, lie explicitly outside it.

I. INTRODUCTION

Minimal Einstein–Cartan–Holst (ECH) gravity is among the simplest torsionful extensions of general relativity motivated by loop quantum cosmology: promoting the spin connection to an independent field, with torsion sourced algebraically by fermion spin currents, introduces exactly one new parameter (the Barbero–Immirzi parameter γ) and one new coupling scale ($\kappa = 8\pi G \sim M_{\rm Pl}^{-2}$), with no new propagating degree of freedom. In loop-quantized cosmology the resulting bounce replaces the classical singularity with a quantum bounce at Planck-scale density. A recurring question in this framework, and in torsion-based bounce cosmology more generally, is whether the same minimal spin-torsion sector that produces the bounce can also source the observed late-time dark-energy density $\rho_{\Lambda,\rm obs} \approx (2.25\,\mathrm{meV})^4$.

This paper answers that question systematically. We enumerate every distinct mechanism class by which minimal ECH could plausibly connect its bounce-scale torsion sector to a late-time Λ-like density — seven structural "foundations" (parameter-space, symmetry, and naturalness arguments) and six observational branches (CMB, gravitational-wave, and relic-abundance channels) — and show that each is closed by an explicit, individually labeled argument: an amplitude-suppression bound, a naturalness/fine-tuning objection, a decoupling identity, or a classification argument. The resulting fourteen-entry catalog collapses to thirteen distinct mechanism-class constraints (one entry is subsumed by a more complete result) and is presented alongside two amplitude-budget closures — Route 2 (one-loop graviton-sector corrections to the Holst term) and Route 3 (quantum running of the Barbero–Immirzi parameter) — together with an operator-list argument establishing that the governing dimensional bound applies not to one representative operator but to the exhibited six-member local dimension-

* houston@hubify.com

four parity-odd operator spanning list admitted by the minimal-coupling field content under stated construction rules (the list is rank-four, hence redundant by construction; that it *spans* the rule-admitted space is asserted from those rules, not proved by exhaustive symbolic enumeration).

Relation to the companion paper.—This survey builds on, and is a companion to, the paper establishing the algebraic torsion-elimination step and the resulting minimal axial–axial four-fermion contact interaction (Route 1) in the same framework [1]. The division of content between the two papers is as follows. The companion publishes the Cartan elimination and the minimal contact operator, a regulated mean-field NJL check of the Route-1 condensate channel, the zero-spin-branch perturbation-transparency theorem, and a Fierz-rearrangement convention appendix with its released verification script; we cite those results rather than rederiving them, and the short derivations a referee needs in order to verify this survey's quantitative claims from this manuscript alone — the torsion-elimination chain fixing the $-(3\kappa/16)[\gamma^2/(1+\gamma^2)]$ contact coefficient and the R1 finite-density benchmark arithmetic — are additionally carried self-contained in Appendix E, following the same faithful-extraction convention as Appendix D. The companion results that remain cited-only (the mean-field NJL gap-equation analysis, the tensor-sector extension of the transparency theorem, and the R4 spectator-ALP consistency check) are used here only with explicit not-peer-reviewed labeling, and their refereeing is part of the companion's own publication path rather than of this survey's. The present survey contributes the material the companion does not publish: the systematic barrier catalog (Sec. III, Table I, Fig. 1), the Route-2/Route-3 amplitude-budget closures (Sec. IV), and the operator-list argument (Sec. V) with its supporting appendices. On the Route-2/Route-3 mappings the two papers take one consistent position: the companion does not retain the historical one-loop Holst/Immirzi-running mappings to late-time dark energy as results, because the underlying calculations do not derive the required cosmological stress tensor and observable matching [2, 3]; this survey does not reinstate them — Sec. IV treats them as historical candidate routes and closes them as bounded amplitude budgets, without supplying (or needing) the missing stress-tensor derivation.

Contributions.—So that the publishable unit of this survey can be judged in one place: beyond the single-scale NDA argument of Appendix A and the companion's published results, this paper contributes (i) the systematic route↔barrier taxonomy (Table I, Fig. 1) with its explicit three-tier evidentiary classification (Table II); (ii) the operator-list closure of Sec. V / Appendix A 1 — the six-member genuine dimension-4 parity-odd spanning list {O1–O6} with its script-verified reduction identities and its two exact rank-reducing relations (that the list spans the rule-admitted space is asserted from the stated construction rule, Sec. V, not proved); (iii) the integrated Route-3 bound $|\Delta\gamma/\gamma| \approx 1.4\times10^{-6}$ obtained from the Benedetti–Speziale one-loop flow; and (iv) the one-loop-grounded Route-2 amplitude budget. A self-contained statement and proof of the perturbation-transparency theorem (B14), the catalog's sole Tier-I leg, is carried in Appendix D so that it can be refereed from this manuscript.

Sec. II fixes the minimal notation this survey needs. Sec. III presents the barrier catalog: the classification of results by evidentiary strength, the barrier-to-route structure (Fig. 1), the formal table (Table I), and the fourteen catalog entries. Sec. IV derives the Route-2 and Route-3 amplitude closures and ties all four enumerated routes together in a single evidentiary-status table. Sec. V presents the operator-list argument. Sec. VI states explicitly what this survey does not claim. Sec. VII concludes.

II. CONVENTIONS AND SETUP

We work in minimal Einstein–Cartan–Holst (ECH) gravity: general relativity extended by promoting the spin connection to an independent field, with the Holst term $\gamma^{-1}\, e^a\wedge e^b\wedge R_{ab}$ added to the first-order action with Barbero–Immirzi parameter γ. Signs, signature, and index conventions follow the companion paper's setup [1]. We use natural units and $\kappa \equiv 8\pi G$ exactly; in terms of the *reduced* Planck mass $\overline{M}_{\rm Pl} \equiv (8\pi G)^{-1/2} \simeq 2.44\times10^{18}\,\mathrm{GeV}$ this is $\kappa = \overline{M}_{\rm Pl}^{-2}$, while order-of-magnitude prose throughout writes $\kappa \sim M_{\rm Pl}^{-2}$ with the *full* Planck mass $M_{\rm Pl} \equiv G^{-1/2}$ — a deliberate factor-8π abuse (exactly, $\kappa = 8\pi\, M_{\rm Pl}^{-2}$), never a definition.

Three convention flags are fixed here once. First, the imported one-loop results of Shapiro–Teixeira and Benedetti–Speziale (Sec. IV) are quoted in those papers' own gravitational-coupling symbol, written $\tilde\kappa$ below, with $\tilde\kappa^2 \equiv 16\pi G$ — in their notation a single coupling of mass dimension -2, *not* the square of this paper's κ (under this paper's κ, $\kappa^2 = M_{\rm Pl}^{-4}$ carries dimension -4). Every $\tilde\kappa$ below refers to that imported convention. Second, the order-of-magnitude numerics in this survey use the full Planck mass $M_{\rm Pl} \simeq 1.22\times10^{19}\,\mathrm{GeV}$; the reduced-versus-full distinction (2.44×10^{18} vs $1.22\times10^{19}\,\mathrm{GeV}$) is immaterial at the ≥ 58-order margins quoted. Where a specific imported numeric depends on the choice, the usage is stated rather than hidden: the Table II R1 benchmark $\kappa n_\psi^2/\rho_\Lambda \simeq 3.6\times10^{-69}$ is the companion's published value [1], computed with the exact $\kappa = 8\pi G$ (equivalently the reduced-mass form) *and* the companion's dark-energy normalization $\rho_\Lambda \approx (2.3\,\mathrm{meV})^4 \approx 2.8\times10^{-11}\,\mathrm{eV}^4$; under the $\rho_\Lambda = (2.25\,\mathrm{meV})^4$ input used in the Appendix A hierarchy, the same κn_ψ^2 evaluates to 3.9×10^{-69} — identical at the order-of-magnitude precision quoted ($\approx$68 orders below ρ_Λ either way), while the Appendix A hierarchy $M_{\rm Pl}^4/\rho_{\Lambda,\rm obs}$ uses the full mass; each is quoted at order-of-magnitude precision only. Third, the symbol pair (α, M) appears in two roles: in the operator ansatz of Sec. V [Eq. (6)], M denotes the LQG area-gap scale $M_{\text{area-gap}} \sim M_{\text{Pl}}/\sqrt{\gamma}$, while every *numerical* occurrence of the combination α/M in this survey (the Route-2 budget, the R4 channel, and Appendix A) uses the single R4-fitted anchor value $\alpha/M \sim 10^{-21}\,\text{GeV}^{-1}$ (Sec. IV C); only the fitted combination α/M — never α or M separately — enters any quantitative statement, so no conclusion depends on resolving the two roles. Similarly, $\beta(\gamma)$ (always written with its argument) is the Route-2 renormalization-group function, while bare β is the observed birefringence angle.

Torsion in ECH is non-propagating: varying the first-order action with respect to the connection gives an algebraic (non-dynamical) constraint fixing the torsion tensor T^{abc} in terms of the fermion spin current S^{abc}, $T^{abc} = \kappa S^{abc}$. For minimally (totally antisymmetrically) coupled Dirac matter, the spin current is dual to the axial current, $S^{abc} = \frac{1}{4}\varepsilon^{abcd} J^5_d$ with $J^{5\mu} = \bar{\psi}\gamma^\mu\gamma^5\psi$. Substituting the algebraic constraint back into the action eliminates torsion in favor of a genuine four-fermion axial-axial contact interaction with coefficient fixed entirely by κ and γ; the companion paper derives this elimination in full and obtains the resulting minimal contact operator, $-(3\kappa/16)[\gamma^2/(1+\gamma^2)]\,(J^5)^2$, together with its mean-field vacuum-condensate analysis (Route 1 of the four-route enumeration below) [1]. The elimination chain and the resulting coefficient are carried compactly, faithful to the companion, in Appendix E; this survey begins from that result and extends the same setup to the remaining routes and to the barrier catalog that constrains all of them.

We enumerate four minimal-ECH channels through which the bounce-scale torsion sector could in principle source late-time dark energy: R1 (the NJL-type four-fermion contact interaction just described), R2 (one-loop graviton-sector corrections to the Holst term), R3 (quantum running of the Immirzi parameter γ), and R4 (a direct parity-odd coupling between the electromagnetic field and a spectator axion-like field or the fermion axial current, which also imprints cosmic birefringence). R1 is addressed in the companion paper; R2 and R3 are closed in Sec. IV below; R4 closes by a naturalness rather than an amplitude argument and is summarized, for completeness of the four-route accounting, in Sec. IV C. Its full derivation is outside the scope of this survey.

III. THE BARRIER CATALOG

Before cataloguing the individual barriers we fix their collective status precisely, so that their joint force is not overstated. We describe the catalog as *13 distinct mechanism-class constraints* (14 historical entries, with B8 subsumed by B14). Here 'distinct' and 'mechanism-class' mean that no barrier is a logical *consequence* of another and each probes a separate physical failure mode (amplitude suppression, thermal washout, operator decoupling, naturalness deficit, and so on); they do *not* assert that the barriers rest on disjoint assumptions or that each is an independent rigorous no-go theorem. In particular, several barriers share the same phenomenological on-shell scaling ansatz (Appendix A); several—B5 (scale separation), B6 (attractor sensitivity), B7 (parameter immunity), B10 (UV→IR specificity), and B13 (gravitational democracy)—are general naturalness or classification arguments that apply to broad classes of bounce/modified-gravity models rather than sharp ECH-specific calculations; and at least one, B9 (Liouville conservation), is an explicitly *heuristic* closure conditional on stated equilibrium assumptions (no particle production, no entropy injection) that realistic quantum bounces can violate. The catalog is therefore best read as a structured map of failure modes of mixed individual strength—quantitative amplitude bounds, naturalness arguments, and qualitative observations—whose collective value is the systematic coverage of the route space, not a claim that thirteen separately decisive theorems each independently exclude the framework. The two sharp, first-principles derivations behind the catalog are the Route-1 torsion-elimination derivation (a companion-paper result [1]) and the perturbation-transparency theorem (B14; established in the companion paper's zero-spin-branch analysis and carried self-contained in Appendix D). Of these, only the perturbation-transparency theorem enters the closure table (Table II) as a Tier-I leg: the torsion-elimination derivation is itself first-principles, but the Route-1 *closure* it feeds rests on Tier-II+III arguments, which is why the abstract credits exactly one Tier-I rigorous theorem. The remaining entries are constraints of the weaker classes just described and are labeled as such below.

We tested 7 foundation mechanism classes (Foundations A–G) and 6 additional observational channels (Branches H, J, L, M, N, O; the ECH perturbation gates enter through Branch H, whose first-principles closure B14 subsumes B8) for the possibility of connecting the ECH bounce to late-time dark energy. Each test yielded a named structural constraint. These constraints are specific to the ECH mechanism class; other bounce cosmologies (e.g., quintom scenarios) are not subject to them.

Constraint classification.—**Novel results** (Barriers 1, 2, 3, 4, 8, 10, 11, 12, 14): constraints first formulated within the ECH analysis, not immediate consequences of prior literature. Novelty here is a *provenance* label, not a rigor or specificity claim: eight of the nine are ECH-specific calculations, while B10 is a general naturalness dilemma first articulated in this catalog in the ECH-bridge context — consistent with its classification among the general naturalness arguments in the preamble above, in its own entry tag, and in Sec. VI. **Known results** (Barriers 5, 6, 7, 9): scale separation, attractor-sensitivity dilemma, parameter immunity, Liouville conservation (B9; heuristic ordering argument)—included to close mechanism classes that arise naturally in the ECH analysis. **Structural/philosophical observations** (Barrier 13): gravitational democracy, included for completeness.

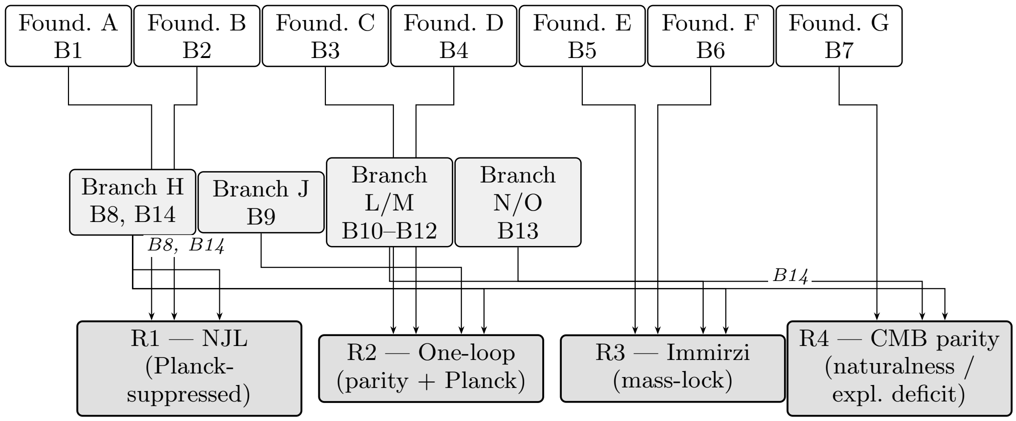

FIG. 1. **Structure of the 14-entry barrier closure.** Top row: the 7 foundation mechanism classes (Foundations A–G; Barriers 1–7, mechanism-class barriers). Middle row: the 6 observational-channel branches (Branches H, J, L, M, N, O; Barriers 8–14, observational-channel barriers), grouped into four boxes — Branch H (B8, B14), Branch J (B9), Branches L/M (B10–B12), and Branches N/O (B13). Bottom row: the four closed dark-energy routes R1–R4; all four routes are closed, giving 13 distinct mechanism-class constraints. Arrows indicate which barrier classes constrain each route. Branch H carries arrows to all four routes; the italic in-figure edge labels give its per-barrier attribution directly: the upper Branch-H arrow to R1 is driven by B8 and B14 together (label "B8, B14"), while the lower fan to R2, R3, and R4 is driven by B14 (perturbation transparency) alone (label "B14"). The branch letters are inherited unchanged from the historical catalog, which never assigned the letters I or K; no branch entries have been removed. B14 (the ECH perturbation-gate closure) sits in Branch H and subsumes B8 (parity-even interaction), so B8 is not counted separately; several entries share the scaling ansatz but each probes a distinct physical failure mode. The diagram records channel structure only, not evidentiary weight: the entries differ in evidentiary status — the sole Tier-I rigorous theorem is B14's perturbation transparency (Appendix D), the remainder are structural or ansatz-level arguments of mixed individual strength, and five entries (B5–B7, B10, B13) are general naturalness or classification arguments — with the per-route classification recorded in Table II.

TABLE I. The 14 catalogue entries (13 distinct mechanism-class constraints; B8 subsumed by B14) on minimal ECH dark-energy routes. Note: Barriers 8 (parity-even interaction) and 14 (perturbation transparency) close the same observable channel (primordial-GW chirality / tensor parity violation) via related routes sharing the perturbation-transparency result; B14 is the first-principles theorem that subsumes B8 as the corresponding observational consequence. They are listed separately to preserve the historical mechanism-class catalog, but should not be counted as a separate mechanism-class constraint.

#	Barrier	Source	Mechanism Blocked
1	Mass-Coupling Lock	Found. A	Propagating torsion as DE
2	Topological-Shift Duality	Found. B	Geometric pseudoscalar mass protection
3	Scalar-Tensor Universality	Found. C	Distinctive geometric content on FRW
4	Planck Suppression	Found. D	Disformal / connection coupling effects
5	Scale Separation	Found. E	Global vacuum integral coupling
6	Attractor-Sensitivity Dilemma	Found. F	Initial-condition transfer to DE
7	Parameter Immunity	Found. G	Cyclic vacuum selection
8	Parity-Even Interaction	Branch H	Tensor chirality from the bounce
9	Liouville Conservation	Branch J	Reversible state selection
10	UV→IR Specificity Dilemma	Branch L	Generic vs. bounce-specific bridge
11	Decoupling Universality	Branch L/M	Light gauge field coupling
12	Vacuum Amplification Ceiling	Branch M	Gravitational wave amplitude
13	Gravitational Democracy	Branch N/O	Relics, baryogenesis, vacuum transitions
14	Perturbation Transparency	Branch H (ECH gates)	ECH-specific perturbation signatures

A. Barrier details

The fourteen catalog entries of Table I are stated compactly below; the route each barrier constrains is given in brackets (the barrier→route map is drawn in Fig. 1). The entries are compact mechanism statements; several (B1, B4, B12) include explicit order-of-magnitude scaling relations inline.

B1 — Mass-Coupling Lock (Found. A) [R1]. In Poincaré gauge theory (PGT), ultralight torsion modes ($m_T \sim H_0$) require $g_{\rm eff} \sim 1/(M_{\rm Pl}\sqrt{|t_3|}) \sim H_0/M_{\rm Pl} \sim 10^{-61}$, where t_3 is the quadratic-torsion coupling controlling the tensor-torsion mode mass, with $\sqrt{|t_3|} \sim m_T^{-1}$ for an ultralight mode (a labeled scaling ansatz of the

PGT mass spectrum, not a derived equality). Achieving $g_{\rm eff} \sim 1$ needs $m_T \sim M_{\rm Pl}$; keeping $m_T \sim H_0$ against a radiative contribution $\delta m_T^2 \sim M_{\rm Pl}^2$ instead requires a cancellation to one part in $(M_{\rm Pl}/H_0)^2 \sim 10^{122}$ — a residual $m_T^2/\delta m_T^2 \sim (H_0/M_{\rm Pl})^2 \sim 10^{-122}$ — the standard cosmological-constant hierarchy.

B2 — Topological-Shift Duality (Found. B) [R1]. In metric-affine gravity, mass protection $\iff$ no geometric fingerprint: configurations protecting the pseudoscalar mass through topological structure eliminate the geometric content (the field reduces to a standard ALP after torsion elimination), while configurations preserving geometric content cannot protect the mass.

B3 — Scalar-Tensor Universality (Found. C) [R2]. On an FRW background, diffeomorphism invariance forces the torsion fluctuation to couple to the curvature invariants like any other scalar, with no ECH-specific observable; torsion decouples from the FRW background precisely at the bounce density, yielding no distinctive perturbation signal.

B4 — Planck Suppression (Found. D) [R2]. Disformal torsion couplings are Planck-suppressed by $m_\phi^2/M_{\rm Pl}^2$ or $(\partial\phi)^2/M_{\rm Pl}^4$; at cosmological scales ($m_\phi \sim H_0$) these are $\mathcal{O}(10^{-122})$—observationally inaccessible.

B5 — Scale Separation (Found. E) [R3]. The global vacuum integral $\int d^4x\,\sqrt{-g}\,\rho_\Lambda$ cannot be connected to the local bounce density without a mechanism to store and transfer the integrated vacuum energy across $N_{\rm tot} \approx$ 92–94 e-folds of inflation (Appendix A); none exists within minimal ECH. (General naturalness/classification argument, not an ECH-specific calculation.)

B6 — Attractor-Sensitivity Dilemma (Found. F) [R3]. If post-bounce inflation converges to an attractor, bounce initial conditions are washed out; if it is sensitive to them, inflation destabilizes. The bounce cannot simultaneously seed dark energy *and* preserve standard inflation dynamics. (General naturalness argument.)

B7 — Parameter Immunity (Found. G) [R4]. Cyclic vacuum selection requires γ to vary across cycles, but γ is fixed by the LQG area spectrum at a universal value; LQG provides no landscape of γ values for selection to operate on. (General classification argument.)

B8 — Parity-Even Interaction (Branch H). *[R1; subsumed by B14.]* The spin-torsion interaction $(J^5)^2$ is parity-*even* (a product of two axial currents is a Lorentz scalar, not a pseudoscalar), so it cannot generate tensor chirality in primordial gravitational waves; independently confirmed by the perturbation-transparency result (B14).

B9 — Liouville Conservation (Branch J) [R2]. Phase-space volume conservation prevents irreversible selection among post-bounce states, closing the "vacuum selection at the bounce" class: the time-symmetric bounce selects no net dark-energy state. This is a *heuristic* closure under explicit assumptions (closed non-dissipative evolution, no particle production, no entropy injection); dissipative/particle-producing bounces evade B9 and are constrained instead by B10–B11, so B9 is never used as a stand-alone closure.

B10 — UV→IR Specificity Dilemma (Branch L) [R3]. Any bridge from Planck-scale bounce physics to the late-time H_0 scale must be either generic (explaining *any* vacuum energy) or bounce-specific (requiring free parameters equivalent to the cosmological constant); no ECH mechanism achieves both. (General naturalness argument.)

B11 — Decoupling Universality (Branches L/M) [R3]. At low energies all gauge fields decouple from the Planck-suppressed torsion sector equally; the ECH bounce cannot preferentially couple to photons or dark-energy degrees of freedom without new non-minimal couplings.

B12 — Vacuum Amplification Ceiling (Branch M) [R3]. Gravitational-wave production from the ECH bounce is bounded above by $\Omega_{\rm GW}^{\rm ECH}|_{\rm bounce} \lesssim (\rho_{\rm crit}/\rho_{\rm Pl})^2 \simeq$ 0.07–0.17, using the LQG-bounce critical-density window $\rho_{\rm crit}/\rho_{\rm Pl} \simeq$ 0.27–0.41: the 0.41 endpoint is the canonical effective-LQC value quoted by Ashtekar–Singh at the standard area-gap choice $\gamma = 0.2375$ [4], while the 0.27 endpoint substitutes the SU(2) black-hole-entropy value $\gamma \approx 0.274$ (the improved state-counting value of Ghosh & Mitra [5]) into the same $\rho_{\rm crit} = \sqrt{3}/(32\pi^2\gamma^3)\,\rho_{\rm Pl}$ formula — an internal extrapolation across entropy-counting schemes established in the companion paper [1], not a value quoted in Ref. [4] — so the window is scheme-dependent rather than a published LQC range. The quadratic scaling is an order-of-magnitude ceiling *ansatz* (not derived here); B12 is used only as a global ceiling. This bounce-epoch fraction is not directly comparable to the present-day PTA spectral density $\Omega_{\rm GW}(f_{\rm nHz}) \sim 10^{-9}$ (which differs by redshift/dilution and by frequency-integration); a quantitative NANOGrav comparison requires propagating the bounce GW spectrum through the transfer function to the nHz band, deferred to a dedicated bounce-GW paper, so B12 closes as a global energy-density-fraction ceiling, not a direct NANOGrav exclusion.

B13 — Gravitational Democracy (Branches N/O) [R4]. Torsion couples democratically to all spin-1/2 species and cannot preferentially source baryogenesis, dark-matter relics, or vacuum transitions without species-dependent non-minimal couplings. (Structural/philosophical observation, included for completeness.) B13 is logically independent of B11 under the criterion of Sec. III ("no barrier is a logical consequence of another"): B11 states that the *gauge* sector decouples universally, which by itself says nothing about relative couplings *among fermion species*, and B13 states that the *matter* sector couples democratically, which by itself permits a gauge sector with species-blind but channelselective couplings. Neither implies the other, and neither is implied by B4 (Planck Suppression), which fixes the *magnitude* of the coupling both invoke but not its *universality* across sectors. They are counted separately for that reason.

B14 — Perturbation Transparency (Branch H, ECH perturbation gates). *[R1–R4.]* For canonical scalar matter, torsion vanishes at all perturbation orders and the Holst sector decouples from all scalar/tensor perturbation observables. The statement and proof are carried self-contained in Appendix D; the tensor-sector extension and the explicit second-order Holst-term verification are elaborated in the companion paper's zero-spin-branch analysis [1]. This is the catalog's sole Tier-I closure leg (Table II).

IV. THE ROUTE-2/ROUTE-3 DARK-ENERGY ROUTE CLOSURES

The two routes closed in this section are *historical* candidate mappings — one-loop Holst-sector corrections (Route 2) and Immirzi-parameter running (Route 3) mapped onto a late-time dark-energy density — that the companion paper explicitly does not retain as results, for the reason recorded in Sec. I. Nothing in this section reinstates them. The closures below are upper-bound *amplitude budgets* demonstrating that any completion of these mappings falls short of the observed signal in its channel by tens of orders of magnitude — for Route 2 the budget is evaluated against the *observed birefringence amplitude* [Eq. (2)], for Route 3 against the *observed dark-energy density* — which is precisely why the catalog closes the corresponding channels. The published one-loop inputs enter as normalization anchors for the budgets, not as derivations of a cosmological observable. We state the division of labour once, at the head of the section, because the two routes are closed against different observables. Every quantitative object in Sec. IV A is a CMB rotation angle: what is computed there is the closure of Route 2's *birefringence* channel. Route 2's *dark-energy* closure is not computed in that subsection; it is inherited from the operator-list argument of Sec. V and the single-scale NDA bound of App. A, which bound every local dimension-four parity-odd density admitted by the minimal-coupling field content. Route 3, by contrast, is bounded directly against $\rho_{\Lambda,\mathrm{obs}}$ in Sec. IV B.

A. Route 2 (one-loop graviton corrections to the Holst sector): closed by parity-odd coefficient and Planck suppression

At the classical level the Holst term $\gamma^{-1}\, e^a \wedge e^b \wedge R_{ab}$ vanishes identically in the torsion-free sector by the first algebraic Bianchi identity (it is *not* a topological invariant in the sense of the Pontryagin density; it reduces to the Nieh–Yan density plus a torsion-squared piece, both of which vanish at $T = 0$, as established in the companion paper's zero-spin-branch analysis [1]) and reduces to the Nieh–Yan density on shell once torsion is integrated out [6, 7]. Quantum corrections from minimally coupled fermions, however, generate a parity-odd coupling between the gravitational field and the chiral fermion current at one loop, with the Holst sector developing running couplings analyzed via renormalization-group methods in Einstein–Cartan + Holst gravity [2, 8]. Motivated by (but *not literally derived in*) the Holst+non-minimal-fermion construction of Mercuri and the one-loop analysis of Shapiro & Teixeira—those works establish the classical structure of the Holst term coupled to fermions, the Nieh–Yan invariant, and the one-loop running of the Holst sector, not this exact effective operator—we adopt the phenomenological one-loop parity-odd operator

$$\Gamma^{\text{parity-odd}}_{\text{one-loop}} = -\frac{1}{16\pi^2}\,\frac{\beta(\gamma)}{M_{\mathrm{Pl}}} \int d^4x\sqrt{-g}\,\partial_\mu \vartheta_{\mathrm{NY}}(x)\, J^{5\mu}, \tag{1}$$

where $\vartheta_{\mathrm{NY}}(x)$ is the Nieh–Yan pseudoscalar (mass dimension $+1$; written ϑ_{NY} to distinguish it from a photon-sector spectator ALP θ of the R4 channel, Sec. IV C; no identification between the two fields is assumed anywhere in this paper), $J^{5\mu}$ is the fermion axial current, and $\beta(\gamma)$ is a slowly varying function of γ. The parity classification of Eq. (1) — the operator is intrinsically parity-even; the parity violation is background-induced — and the anomaly-mediated photon chain used in the dimensionless ratio below are stated in Appendix B. The function $\beta(\gamma)$ is written with its explicit argument throughout to distinguish this renormalization-group function from the cosmic birefringence angle β of the R4 channel (Sec. IV C). The coefficient structure of Eq. (1) is *grounded in the explicit one-loop computation* of Shapiro & Teixeira [2], who renormalize on-shell Einstein–Cartan+Holst gravity with external vector and axial fermion currents. The parity-odd, Immirzi-dependent current coupling they renormalize is $\lambda_4 = \gamma\, \tilde\kappa^2\, (W \cdot J)$ (their Eq. 37; $\tilde\kappa^2 \equiv 16\pi G \sim M_{\mathrm{Pl}}^{-2}$, the imported convention fixed in Sec. II), with classical coefficient $\alpha_4 = -6/(1+\gamma^2)$ (their Eq. 41) and one-loop divergence coefficients $\Omega_{44} = -(378+783\gamma^2)/[20(1+\gamma^2)^2]$, $\Omega_{24} = -81\gamma^2/(1+\gamma^2)^2$ (their Eq. 42, arXiv version; the λ_4 flow, their Eq. 51, is driven by the $\Omega_{44}, \Omega_{24}, \Omega_{34}$ family). Their master renormalization-group equation $d\lambda/dt = -\sigma/(4\pi)^2$ (their Eq. 46) carries *exactly* the $1/(16\pi^2)$ loop factor already written in Eq. (1). Three features of the Route-2 budget are therefore fixed by a published one-loop result rather than adopted: (i) the loop factor $1/(16\pi^2)$; (ii) the coefficient $\beta(\gamma)$ is an $\mathcal{O}(1)$ rational function of the Immirzi parameter [the α_4/Ω_{4x} family, e.g. $|\Omega_{44}/\alpha_4| = (378+783\gamma^2)/[120(1+\gamma^2)]$, which is $\approx$3.3 at the LQG value $\gamma \approx 0.24$ and $\mathcal{O}(3)$–$\mathcal{O}(5)$ across $\gamma \lesssim \mathcal{O}(1)$ (the ratio is bounded below by $378/120 \approx 3.2$ for all real γ)], not a free normalization; and (iii) the explicit $\tilde\kappa^2 \sim M_{\mathrm{Pl}}^{-2}$ on every renormalized charge, i.e. the Planck suppression in the prefactor $\beta(\gamma)/M_{\mathrm{Pl}}$. What the one-loop analysis does *not* fix is the single *absolute* normalization: Shapiro & Teixeira show the coupled flow for $\lambda_4(t)$ and $\gamma(t)$ (their Eqs. 51, 58) is a Riccati system whose particular-solution roots are complex for real γ, so the Shapiro–Teixeira λ_4–γ system *has no renormalization-group fixed point* (in contrast to the distinct Benedetti–Speziale flow of Route 3 below, which has one) and they were "unable to solve it in a completely satisfactory way." A fully-derived Route-2 amplitude would require a UV boundary condition plus a controlled solution of that non-perturbative flow (or a dedicated matching calculation for the exact $\partial_\mu\vartheta_{\rm NY}J^{5\mu}$ operator). We therefore treat the *absolute* $\beta(\gamma)$ normalization as a bounded EFT input while the loop factor, Immirzi-rational coefficient, and Planck suppression are one-loop-grounded. This is strictly weaker than the Route-3 result below (whose Benedetti–Speziale β-function integrates cleanly to $|\Delta\gamma/\gamma| \approx 1.4\times10^{-6}$) but strictly stronger than a free ansatz: the residual freedom is a single $\mathcal{O}(1)$ normalization. Route 2's *dark-energy* closure, as distinct from the birefringence closure computed in this subsection, is not carried by that normalization at all. It is inherited from the operator-list argument of Sec. V and the single-scale NDA no-go of App. A, which bound every local dimension-four parity-odd density admitted by the minimal-coupling field content — the class into which Eq. (1) falls — regardless of that $\mathcal{O}(1)$ coefficient, and ST's explicit $\tilde\kappa^2 \sim M_{\rm Pl}^{-2}$ confirms the missing powers are Planck powers, not a light scale. (The dimension-(+1) object bounded directly in App. A is the phenomenological representative Eq. (7), not Eq. (1), which as shown below already carries dimension +4 with no deficit.) Because the closure below retains ≈ 60 (conservatively ≥ 58) orders of margin, it is insensitive to this residual $\mathcal{O}(1)$ ambiguity in $\beta(\gamma)$. The prefactor is $\mathcal{O}(\alpha_{\rm em}/4\pi)$ — a dimensionless loop factor — times a single inverse power of the Planck mass; the product carries mass dimension -1 and is not itself dimensionless. Explicitly, the prefactor $\beta(\gamma)/M_{\rm Pl}$ carries mass dimension -1 (the division by $M_{\rm Pl}$, not multiplication): with $\vartheta_{\rm NY}$ of dimension +1, $\partial_\mu\vartheta_{\rm NY}$ has dimension +2 and the axial current $J^{5\mu} = \bar\psi\gamma^\mu\gamma^5\psi$ has dimension +3, so the full term — the dimension-(+5) integrand $\partial_\mu\vartheta_{\rm NY}J^{5\mu}$ times the dimension-(-1) prefactor $\beta(\gamma)/M_{\rm Pl}$ written outside the integral in Eq. (1) — carries dimension $-1+2+3 = +4$ and the action is dimensionless, as required. Once the dimension-(+2) background value $\partial_\mu\vartheta_{\rm NY} \sim H_0^2 \sim 10^{-66}\,{\rm eV}^2$ at the present epoch is substituted (the cosmological Nieh–Yan pseudoscalar, of dimension +1 and Hubble-scale amplitude, evolving on the Hubble time), the one-loop rotation accumulated between recombination and today — over an elapsed time of order the Hubble time, $\Delta t \sim H_0^{-1}$ — assembles from the stated ingredients as the explicit intermediate estimate

$$\begin{aligned}\Delta\theta_{\text{one-loop}} &\sim \frac{\alpha_{\rm em}}{4\pi}\,\frac{\partial_\mu\vartheta_{\rm NY}}{M_{\rm Pl}}\,\Delta t\\ &\sim \frac{\alpha_{\rm em}}{4\pi}\,\frac{H_0^2}{M_{\rm Pl}}\,H_0^{-1} = \frac{\alpha_{\rm em}}{4\pi}\,\frac{H_0}{M_{\rm Pl}}\end{aligned}$$

(the anomaly-mediated photon chain of Appendix B supplies the $\alpha_{\rm em}/4\pi$; the one-loop factor $1/16\pi^2$ and the $\mathcal{O}(1)$ coefficient $\beta(\gamma)$, each of which could only suppress the estimate further, are conservatively dropped — this Tier-III accumulation ansatz is exactly the bookkeeping the evidentiary label of Table II records), and is stated as the dimensionless budget ratio

$$\begin{aligned}\frac{\Delta\theta_{\text{one-loop}}}{\beta_{\rm obs}\left[M_{\rm Pl}(\alpha/M)\right]} &\sim \frac{\alpha_{\rm em}}{4\pi}\,\frac{H_0/M_{\rm Pl}}{M_{\rm Pl}\,(\alpha/M)\,\beta_{\rm obs}}\\ &\sim \frac{\alpha_{\rm em}}{4\pi}\,\frac{H_0}{M_{\rm Pl}}\,\frac{M}{\alpha\,M_{\rm Pl}\,\beta_{\rm obs}}\,, \end{aligned} \tag{2}$$

whose left-hand side normalizes the one-loop amplitude *both* by the observed angle $\Delta\theta_{\rm obs} = \beta_{\rm obs}$ *and* by the dimensionless strength $M_{\rm Pl}(\alpha/M) \sim 10^{-2}$ of the R4-fitted coupling that reproduces that angle: the double normalization states the suppression relative to the fitted-coupling benchmark, the weaker (more conservative) of the two available normalizations, and both normalizations are evaluated explicitly below. Here $\alpha_{\rm em}/(4\pi) \approx 6\times10^{-4}$ (more precisely 5.8×10^{-4}, using the Thomson-limit fine-structure constant $\alpha_{\rm em} \simeq 1/137$; a running $\alpha_{\rm em}(\mu)$ evaluated at any higher scale differs by an $\mathcal{O}(1)$ factor immaterial at these margins), rounded *up* to 10^{-3} in the explicit order-of-magnitude substitutions below — an overestimate of the numerator and hence conservative for the suppression claim; the closure is robust to this factor. Here $H_0/M_{\rm Pl} \sim 10^{-61}$, and the R4-fitted coupling $\alpha/M \sim 10^{-21}\,{\rm GeV}^{-1}$ (Sec. IV C) gives $M_{\rm Pl}\cdot(\alpha/M) \sim 10^{19}\,{\rm GeV}\cdot10^{-21}\,{\rm GeV}^{-1} = 10^{-2}$. Plugging in $\beta_{\rm obs} = 0.342^\circ \approx 6\times10^{-3}\,{\rm rad}$ ($0.342\times\pi/180 = 5.97\times10^{-3}$), the dimensionless ratio Eq. (2) evaluates to $10^{-3}\cdot10^{-61}/(10^{-2}\cdot6\times10^{-3}) \approx 10^{-60}$ (canonical evaluation of the displayed contraction; we conservatively allow up to two orders of magnitude for unmodeled higher-order loop-ordering corrections, i.e. suppression by at least 10^{-58} — an explicit conservatism allowance, *not* a derived range; the eV-vs-GeV unit conversion is exact $1\,{\rm GeV} = 10^9\,{\rm eV}$ and is not a source of ambiguity). Contracting directly against the observed angle alone gives $(\alpha_{\rm em}/4\pi)(H_0/M_{\rm Pl})/\beta_{\rm obs} \approx 10^{-3}\cdot10^{-61}/(6\times10^{-3}) \approx 2\times10^{-62}$, *two additional orders* of suppression; the quoted $\sim10^{-60}$ is therefore the conservative side of this bookkeeping choice, and the closure holds a fortiori under either contraction. Thus the one-loop induced β is suppressed by $\approx$60 (conservatively ≥ 58) orders of magnitude relative to the observed signal. This is the survey's one authoritative Route-2 robustness statement: even inflating the one-loop coefficient by ten orders of magnitude leaves $\gtrsim 48$ orders of suppression margin, so no plausible $\mathcal{O}(1)$–$\mathcal{O}(10^{10})$ normalization ambiguity reopens the route. We adopt this contraction as the canonical Route-2 estimate; an alternative ordering that contracts the H_0 factor with the dimensionful coupling differently yields only a far looser upper bound and is not used in the closure. The canonical-bound conclusion that the one-loop induced β is amplitude-suppressed many orders of magnitude below the observed WMAP+Planck birefringence signal (with ACT DR6 follow-up) is robust to this choice; Route 2 remains exploratory framing, not load-bearing for the no-go. The Route-2 amplitude is therefore far below not only the WMAP+Planck birefringence sensitivity but the observed central value itself; the one-loop Holst-sector parity-odd term cannot account for the observed birefringence amplitude. (A naive comparison of a rotation rate $\dot{\beta}$ in eV against an angle uncertainty in eV would silently treat eV·s as dimensionless; the dimensionless reduction above avoids this and recovers the standard R2 amplitude-suppression closure.) *Closure: amplitude-suppressed by* $M_{\rm Pl}^{-1}$ *and one-loop factor* $\alpha_{\rm em}/(4\pi)$. The evidentiary status of the estimate, stated once: Eq. (1) is an *illustrative upper-bound amplitude budget* constructed as the natural EFT operator at the $M_{\rm Pl}^{-1}\alpha_{\rm em}/(4\pi)$ scale, *not* a result extracted from Mercuri [8] or Date–Kaul–Sengupta [9], which establish the classical Holst/Nieh–Yan structure but not this coefficient; it is presented as an amplitude-budget bound, not a rigorous derivation, and the same logic applies to Route 3 below.

B. Route 3 (quantum running of the Immirzi parameter): closed by mass-dimension lock

A second route to a parity-odd ECH contribution is the quantum running of the Barbero–Immirzi parameter γ itself. Date, Kaul & Sengupta analyzed the Holst term coupled to fermions and the Nieh–Yan invariant in the chiral-matter setting [9]; that analysis establishes a topological interpretation of γ and motivates a γ-running in the presence of chiral asymmetry, but does *not* itself present the explicit RG equation used below. Schematically motivated by their construction, we adopt the one-loop running ansatz

$$\frac{d\gamma}{d\ln\mu} = \frac{1}{12\pi^2}\,(N_F^L - N_F^R)\,\gamma + \mathcal{O}(\gamma^2), \tag{3}$$

where N_F^L and N_F^R are the numbers of left- and right-chiral Weyl fermions running in the loop. The $1/(12\pi^2)$ prefactor is the natural chiral-loop coefficient at this order; we use Eq. (3) only as an upper-bound EFT ansatz for the Route-3 amplitude budget and do not claim it is taken verbatim from [9]. The actual fermion-induced perturbative running of the Immirzi parameter is computed by Benedetti & Speziale [10], who find a β-function whose sign depends on $|\gamma|$ through four-fermion interactions generated when fermions are coupled to the Holst sector; our Eq. (3) is a chiral-count EFT bound rather than the full perturbative result, and is used solely for the amplitude budget below. We can, moreover, do better than an estimate. The actual fermion-coupled one-loop β-function was computed by Benedetti & Speziale; we quote it from their proceedings summary [3] (whose Eq. 7 is the equation numbering used here), companion to the full one-loop analysis of Ref. [10]:

$$\mu\,\frac{\partial\gamma^2}{\partial\mu} = -(\gamma^2{-}1)\,\frac{\mu^2\tilde{\kappa}^2}{(8\pi)^2}\,(23\gamma^2{+}5), \qquad \tilde{\kappa}^2 \equiv 16\pi G, \tag{4}$$

whose only fixed point (of this Benedetti–Speziale flow; the fixed-point-free Riccati system above is Shapiro–Teixeira's Route-2 λ_4–γ system, a different flow) is the ultraviolet-attractive $\gamma^2 = 1$ (formally outside perturbative control), with the sign of the flow set by whether $|\gamma| \gtrless 1$ and the running driven by the radiatively-generated four-fermion interaction. The decisive feature is the explicit $\mu^2\tilde{\kappa}^2 \simeq (\mu/M_{\rm Pl})^2$ prefactor: the running is *power-suppressed* by the renormalization scale in Planck units, not logarithmic, so the accumulated $\int\beta_{\gamma^2}\,d\ln\mu \propto \int\mu\,d\mu$ is dominated by the ultraviolet endpoint and is of order $(\mu_{\rm UV}/M_{\rm Pl})^2$. Integrating Eq. (4) numerically from a GUT-scale ultraviolet boundary $\mu_{\rm UV} \sim 10^{16}$ GeV down to $\mu_{\rm IR} \sim 1$ GeV (with γ near the LQG value $\gamma \approx 0.24$) gives, in agreement with a frozen-coefficient analytic estimate, $|\Delta\gamma/\gamma| \approx 1.4\times10^{-6}$ (evaluated with the full $M_{\rm Pl} \simeq 1.22\times10^{19}$ GeV, per the Sec. II convention for order-of-magnitude numerics). This is a genuine integrated running, *not* an ansatz bound: the explicit $\mu^2\tilde{\kappa}^2 \simeq (\mu/M_{\rm Pl})^2$ prefactor makes the flow power-suppressed, so $|\Delta\gamma/\gamma| \sim (\mu_{\rm UV}/M_{\rm Pl})^2$ and the integral is dominated by the ultraviolet endpoint. The result is perturbatively controlled for any sub-Planckian UV boundary; only a literal Planck-scale boundary would push $|\Delta\gamma/\gamma| \to \mathcal{O}(1)$, precisely the regime Benedetti & Speziale flag as outside perturbative control (the $\gamma^2 = 1$ fixed point corresponds to a divergent four-fermion coupling), so a Planck-scale evaluation would require UV-completion input beyond the one-loop β-function. The derived physical running therefore only *strengthens* Route 3, and this GUT-UV integrated value is the primary Route-3 estimate. Propagated to the dark-energy channel through the mass-dimension scaling relation Eq. (5) displayed below, the derived torsion/Immirzi contribution to ρ_Λ sits $\sim$ 61–67 orders of magnitude below the observed dark-energy density $\rho_{\Lambda,\rm obs} \approx (2.25\,\text{meV})^4$ — so the no-go closes with enormous margin, now from a *derived* integrated result rather than an ansatz upper bound. We nonetheless retain the far larger chiral-count estimate Eq. (3), $\Delta\gamma/\gamma \sim 0.3$, as a deliberately pessimistic *upper* bound in the budget below. In the Standard Model, the chiral asymmetry is generated by the $SU(2)_L$ doublets. Integrating Eq. (3), which is linear in γ, gives $\Delta\ln\gamma \approx (N_F^L - N_F^R)\,\ln(\mu_{\rm GUT}/\mu_{\rm IR})/(12\pi^2)$; with a net chiral count $(N_F^L - N_F^R) = \mathcal{O}(1)$ and a GUT-to-IR lever arm $\ln(\mu_{\rm GUT}/\mu_{\rm IR}) \approx 30$–$37$ ($\mu_{\rm GUT} \sim 10^{16}$ GeV throughout: the upper edge from running down to $\mu_{\rm IR} \sim 1$ GeV, $\ln 10^{16} \approx 36.8$; the lower edge from cutting the flow at $\mu_{\rm IR} \sim 1$ TeV, i.e. crediting only running above colliderprobed scales, $\ln 10^{13} \approx 30$), this is numerically $\Delta \ln\gamma \approx 0.25$–$0.31$ (a few $\times 10^{-1}$; e.g. $32/(12\pi^2) \approx 0.27$; we identify $\Delta\gamma/\gamma \simeq \Delta\ln\gamma$ at this order — exponentiating would give 0.29–0.36, a distinction immaterial at the $\gtrsim$ 60-order margins below). We therefore adopt the larger $\Delta\gamma/\gamma \sim 0.3$ as the *conservative* (least-suppressed) order-of-magnitude estimate — *not* a precisely derived value, and *not* claimed to be structurally consistent with the Benedetti–Speziale flow. The two flows are structurally different: Eq. (3) is purely logarithmic with its only fixed point at $\gamma = 0$, whereas Eq. (4) is power-suppressed by $\mu^2\tilde{\kappa}^2$ with its sole fixed point at $\gamma^2 = 1$ and sign set by $|\gamma| \gtrless 1$. What is defensible, and all that is used, is the *numerical* inequality $0.3 \gg 1.4 \times 10^{-6}$: the chiral-count value numerically bounds the derived integrated running from above — and note that the Route-3 closure below is insensitive to its precise value: even this $\mathcal{O}(0.3)$ running is $\ll 1$, so γ retains its order of magnitude, and it still leaves $\gtrsim 60$ orders of suppression margin. The Holst sector amplitude that this running can source is fixed by mass dimension: any operator built from γ, R_{ab}, e^a, and the chiral current $J^{5\mu}$ must carry dimension four, which forces a single power of $M_{\rm Pl}^{-1}$ in the prefactor in any cosmologically relevant scalar-curvature regime. Writing that scaling out explicitly, the induced parity-odd contribution to the late-time vacuum energy is

$$\frac{\rho_{\rm R3}}{\rho_{\Lambda,\rm obs}} \sim \left(\frac{\Delta\gamma}{\gamma}\right)\left(\frac{H_0}{M_{\rm Pl}}\right), \tag{5}$$

where the reference budget in the denominator is the observed dark-energy density itself, $\rho_{\Lambda,\rm obs} \approx (2.25\,\mathrm{meV})^4$: the single inverse Planck power fixed by the dimension count is saturated, in a late-time cosmological background, by the only available infrared scale, the present Hubble rate H_0, and $\Delta\gamma/\gamma$ is the dimensionless amplitude the running supplies. Eq. (5) is a Tier-III mass-dimension scaling relation, not a derived stress-tensor matching — we do not supply the cosmological stress tensor that a genuine mapping would require, and none is claimed — but it is what the "61–67 orders" statement means, and it is stated here rather than left implicit. Numerically, with $H_0/M_{\rm Pl} = 1.18 \times 10^{-61}$: the conservative chiral-count input $\Delta\gamma/\gamma \sim 0.3$ gives $\rho_{\rm R3}/\rho_{\Lambda,\rm obs} \sim 3 \times 10^{-62}$ ($\sim$61 orders), and the derived integrated value $|\Delta\gamma/\gamma| \approx 1.4 \times 10^{-6}$ gives $\sim 1.7 \times 10^{-67}$ ($\sim$67 orders) — the two endpoints quoted in the abstract. Both close this route by many orders of magnitude. Note that H_0, not a bounce-epoch H, is the correct insertion throughout: at the bounce $H/M_{\rm Pl} = \mathcal{O}(1)$, and the claim being made is about the *late-time* dark-energy channel. *Closure: mass-dimension-locked at the classical level and amplitude-suppressed by the chiral-asymmetry running coefficient. Ansatz vs derivation (R2/R3):* R3's magnitude is now a *derived integrated result*: for a sub-Planckian (GUT-scale) UV boundary the verified Benedetti–Speziale one-loop β-function Eq. (4) integrates to $|\Delta\gamma/\gamma| \approx 1.4 \times 10^{-6}$, replacing the prior chiral-count ansatz upper bound as the primary Route-3 estimate. R2's amplitude coefficient is now *one-loop-grounded*: its loop factor $1/(16\pi^2)$, $\mathcal{O}(1)$ Immirzi-rational coefficient, and $\tilde{\kappa}^2 \sim M_{\rm Pl}^{-2}$ Planck suppression are fixed by the Shapiro & Teixeira [2] one-loop renormalization of the Holst+fermion sector (their Eqs. 41–42, 46); only the single absolute normalization remains a bounded EFT input, because the Shapiro–Teixeira λ_4–γ flow has no fixed point and is not perturbatively solvable in closed form, and the operator is bounded by the single-scale NDA no-go regardless of that $\mathcal{O}(1)$ normalization. The kept ansatz value Eq. (3) ($\Delta\gamma/\gamma \sim 0.3$) is retained only as a deliberately pessimistic upper bound; even under it the amplitude suppression is $\sim 3 \times 10^{-62}$ vs ρ_Λ, and the derived 1.4×10^{-6} value only widens that margin. *Strict theoretical limitation:* the NDA one-loop operator (R2) and Immirzi-running (R3) inputs used here are bounded EFT inputs, not UV-complete flow derivations—the R2 absolute normalization requires a controlled solution to the ST Riccati system (no real fixed point; ST explicitly state they could not solve it satisfactorily), and the R3 chiral-count Eq. (3) is an order-of-magnitude ansatz rather than the full Benedetti–Speziale four-fermion-driven RG flow at all loop orders. These represent the strict theoretical limitation of the R2/R3 analysis within current loop-gravity technology; the $\approx$58–60-order (and larger) closure margins make the qualitative conclusions robust to this limitation, but the inputs cannot be promoted to precision derivations without a UV-complete treatment of the coupled Holst+fermion RG system.

C. The fourth channel and the combined closure

The fourth enumerated channel, R4, is a direct parity-odd coupling between the electromagnetic field and a spectator axion-like field or the fermion axial current, $\mathcal{L}_{\rm CS} \supset -\frac{1}{4}(\alpha/M)\,\phi\,\widetilde{F}_{\mu\nu}F^{\mu\nu}$, producing a rotation of the CMB polarization plane (cosmic birefringence). Fitting this coupling to the observed WMAP+Planck birefringence signal $\beta_{\rm obs} = 0.342^\circ \pm 0.094^\circ$ (WMAP+Planck [11]; the first Planck-2018 extraction reported $0.35^\circ \pm 0.14^\circ$ [12]) (comparable to the independent ACT DR6 measurement $\beta = 0.215^\circ \pm 0.074^\circ$ [13]) gives the anchor value $\alpha/M \sim 10^{-21}\,\mathrm{GeV}^{-1}$ used above as a numerical input to the Route-2 amplitude estimate. Both measurements are statistical indications rather than established detections — $\approx$3.6σ for WMAP+Planck and $\approx$2.9σ for ACT DR6, significances obtained from different datasets and distinct null procedures and therefore not directly comparable as statistical weights — and the Route-2 suppression conclusion is insensitive to that status: a smaller or vanishing true birefringence signal only *widens* the margin of Eq. (2). The algebraic origin of that scale, carried from the companion's derivation [1], is the standard small-rotation result $\beta = (\alpha/2M)\,\Delta\phi_{\rm rec\to today}$ (the rotation angle is half the coupling times the coherent field excursion), with excursion $\Delta\phi \sim \sqrt{2\rho_\theta}/m_\theta \sim M_{\rm Pl}$ for a frozen ($m_\theta \lesssim H_0$)

spectator carrying $\rho_\theta \sim \rho_\Lambda$; matching $\beta_{\rm obs} \approx 6\times10^{-3}$ rad then gives $\alpha/M \sim 2\beta_{\rm obs}/M_{\rm Pl} \sim 10^{-21}\,{\rm GeV}^{-1}$. R4 does not close by amplitude suppression: the same coupling that reproduces $\beta_{\rm obs}$ can, with a free normalization, also be tuned to reproduce ρ_Λ, but minimal ECH does not derive either the required ultralight mass $m_\theta \sim H_0$ or the fitted α/M from first principles—so the channel closes as a naturalness/explanatory-deficit objection (it relocates rather than solves the cosmological-constant problem) rather than as an amplitude exclusion. Its full derivation, and the companion consistency check against a spectator-ALP birefringence benchmark, are outside the present survey's scope [1, 14].

Within the four-channel enumeration of Sec. II, Routes R1–R4 cover the four parity-odd or dark-energy channels of this framework. Two operators occasionally raised as omissions from this four-channel enumeration — the Jackiw–Pi gravitational Chern–Simons term $R\wedge\widetilde{R}$ and the parity-odd four-fermion partner of R1 carrying the $\gamma_{\rm BI}/(\gamma_{\rm BI}^2{+}1)\cdot 8\pi G$ coefficient — are closed in the companion analysis [1] (total derivative for constant coupling and R4-class otherwise, for the Jackiw–Pi term; Planck suppression plus vanishing mean field, inheriting R1, for the four-fermion partner); the dimension-6 parity-odd four-fermion basis is Fierz-closed and uniformly Planck-suppressed by the projection lemma of Appendix C, so only a non-minimal completion lies outside the established no-go. Within the four enumerated channels: R1 (NJL contact) is amplitude-suppressed by $M_{\rm Pl}^{-2}$ and parity-even. R2 (one-loop graviton corrections) is amplitude-suppressed by $M_{\rm Pl}^{-1}$ and the one-loop factor $\alpha_{\rm em}/(4\pi)$. R3 (Immirzi running) is mass-dimension-locked and additionally suppressed by the chiral-asymmetry beta function. R4 (parity-odd CMB coupling) is closed by a naturalness objection: with α/M treated as a free parameter, a spectator-ALP fit reproduces both $\beta_{\rm obs}$ and ρ_Λ, but minimal ECH does not derive $m_\theta \sim H_0$ or the fitted α/M, so the channel closes at the level of an explanatory deficit rather than an amplitude exclusion.

Tiered closure structure: R1–R3 achieve *mathematical amplitude closure* (the predicted amplitude is suppressed by explicit $M_{\rm Pl}$ powers and loop factors, placing it many orders below any observable threshold regardless of normalisation assumptions); R4 achieves *naturalness/explanatory closure* (the amplitude can be matched to observations but only at the cost of importing the cosmological-constant fine-tuning the channel was meant to resolve). These are logically distinct closure modes and are classified separately in the evidentiary table below.

a. Evidentiary status of each leg. So that the strength of the closure is not overread, we state explicitly the evidentiary level at which each route and each structural result is established. We use a three-tier scale: **(I) rigorous result** — a deductive consequence of stated equations/identities, holding exactly within an explicitly bounded scope; **(II) structural argument** — a qualitative or order-of-magnitude argument from established physics (symmetry, parity, naturalness) that does not turn on a fitted number; and **(III) ansatz-level dimensional estimate** — an amplitude budget evaluated under an explicitly-labeled scaling ansatz, honest to a factor but not a first-principles derivation. Table II classifies every leg on this scale. No leg is claimed at a level higher than this table records: in particular, the only Tier-I (rigorous) leg is the perturbation-transparency result for canonical scalar matter; R2 is a Tier-III ansatz-level estimate, and the R1/R3 *amplitude* legs are likewise Tier-III (their structural components — the parity-even mean-field fact for R1 and the mass-dimension lock for R3 — are the Tier-II entries Table II records alongside them); and the R4 closure is a Tier-II *naturalness/explanatory-deficit* objection, *not* an amplitude exclusion and conditional on the on-shell scaling ansatz of Appendix A. The aggregate "four-route channel-level closure" is therefore exactly that: a channel-level (not operator-level) statement whose individual legs sit at the levels tabulated, reinforced but not promoted by the mechanism-class catalog of Sec. III.

The phenomenological parameter α/M introduced above is therefore best understood as the R4-bounded coupling required to match $\beta_{\rm obs}$, with the dark-energy density supplied by an unrelated sector (e.g. a quintessence field [15], a quintom-class two-field scenario [16], or by a positive cosmological constant), rather than by ECH-internal physics. This inversion of the prior mass-coupling lock is a core structural finding reused throughout this catalog.

V. THE OPERATOR-LIST ARGUMENT

The barrier catalog and the Route-2/Route-3 closures above bound the amplitude of *one* representative parity-odd operator. A referee may reasonably ask whether the same conclusion survives once the full local operator space is considered. This section answers that question: we exhibit a finite *spanning list* of local, gauge-invariant, diffeomorphism-covariant parity-odd densities of mass dimension exactly four admitted by the minimal-ECH field content under the construction rule stated below, and show that the single-scale dimensional bound applies to every member of that list, not to one representative. We stress at the outset that the list is a generating set, not a linearly independent basis: it carries two exact relations, quantified in Eq. (11) below, and is retained in redundant form because each entry is a separately *recognizable* invariant of the literature. Bounding every member of a spanning list bounds every density it spans, which is all the no-go requires; linear independence is neither claimed nor needed.

As a phenomenological ansatz for a direct parity-odd curvature coupling — motivated by, but not literally derived from, the Holst-plus-non-minimal-fermion con-

TABLE II. Evidentiary status of each closure leg, on the three-tier scale of the text: (I) rigorous result within stated scope, (II) structural argument, (III) ansatz-level dimensional estimate. The table records the highest level at which each leg is claimed; no leg is asserted more strongly elsewhere. "Rules out" is to be read at channel-amplitude granularity, not as an operator-level theorem.

Leg	What it rules out (channel-amplitude level)	Evidentiary status
Perturbation transparency (companion paper [1])	The Holst sector / Barbero–Immirzi parameter contributing to scalar or tensor perturbation observables, for canonical scalar matter	**(I) Rigorous** within stated scope: $T=0$ follows from zero scalar spin density, and the Holst dual contraction vanishes by the algebraic Bianchi identity. Excludes propagating-torsion, fermion-loop, dynamical-Immirzi, non-minimal-matter sectors.
R1 (NJL four-fermion contact)	The torsion-induced contact term sourcing coherent $w = -1$ dark energy	**(II)+(III)**: parity-even structure ($\langle J^5\rangle \approx 0$, no coherent mean field) is a Tier-II algebraic fact; the $M_{\rm Pl}^{-2}$ amplitude suppression is a Tier-III order-of-magnitude estimate from a standard Cartan torsion-elimination derivation: the companion's coefficient-one benchmark $\kappa n_\psi^2/\rho_\Lambda \simeq 3.6\times10^{-69}\,(n_\psi/100\,{\rm cm}^{-3})^2$, i.e. ≈68 orders below ρ_Λ even at a deliberately elevated ISM-like normalization [1] (arithmetic carried self-contained in App. E).
R2 (one-loop graviton corrections)	One-loop promotion of $\gamma_{\rm BI}$ reaching the observed birefringence amplitude	**(III) Ansatz-level**: amplitude budget suppressed by $H_0/M_{\rm Pl}$ and $\alpha_{\rm em}/(4\pi)$ under an explicitly-labeled scaling ansatz; exploratory, not load-bearing.
R3 (Immirzi running)	Quantum running of $\gamma_{\rm BI}$ generating a dark-energy-relevant shift	**(II)+(III)**: mass-dimension lock is structural; the chiral-count ansatz bound (motivated by Date–Kaul–Sengupta [9]) is a Tier-III order-of-magnitude upper bound, deliberately loose and non-load-bearing ($\gtrsim 60$ orders of margin); the integrated Benedetti–Speziale flow gives the far smaller derived primary estimate (Sec. IV).
R4 (parity-odd CMB / spectator-ALP coupling)	Minimal ECH *deriving* the single coupling that yields both $\beta_{\rm obs}$ and ρ_Λ	**(II) Structural naturalness objection**, *not* an amplitude exclusion: a free-α/M ALP fit reproduces $\beta_{\rm obs}$, but ECH supplies neither $m_\theta \sim H_0$ nor the fitted α/M, relocating the CC problem. Conditional on the on-shell scaling ansatz (App. A).

struction of Mercuri [8] — consider the parity-odd term

$$S_{\rm eff} = \frac{\alpha}{M}\int e_I \wedge e_J \wedge \mathcal{F}^{IJ}[K,\mathring{R}], \tag{6}$$

where $M = M_{\text{area-gap}} \sim M_{\rm Pl}/\sqrt{\gamma}$ is the LQG area-gap mass scale (from the LQG area-gap $\Delta \propto \gamma\,\ell_P^2$, the inverse-length / mass scale is $M_\Delta \sim M_{\rm Pl}/\sqrt{\gamma}$ up to numerical constants), α is a dimensionless coupling, and $\mathcal{F}^{IJ}[K,\mathring{R}]$ denotes the curvature two-form of the full (torsionful) connection, written as a functional of the contorsion K and the Levi-Civita curvature $\mathring{R}$; the component form Eq. (7) displays its leading contribution.

In components, the leading contribution reduces to

$$S_{\rm eff} = \int d^4x\,\sqrt{-g}\,\frac{\alpha}{M}\,\varepsilon^{\mu\nu\rho\sigma}\,e^I_\mu\,e^J_\nu\,\mathcal{F}_{IJ\rho\sigma}, \tag{7}$$

which has naive mass dimension $[\mathcal{L}_{\rm odd}] = +1$—three units short of the required $+4$ (with $[\alpha]=0$, $[M]=+1$, $[e^I_\mu]=0$, and $[\mathcal{F}_{IJ\rho\sigma}]=+2$, the integrand carries mass dimension $-1+2=+1$). We state this off-shell mismatch deliberately, and we address it immediately by identifying the physical foundation before using the shorthand.

Physical foundation: the dimension-4 off-shell operator list.—The formally dimension-4, gauge-invariant, diffeomorphism-covariant off-shell operator content is the closed local expansion

$$S_{\rm eff}^{(4)} = \int d^4x\,\sqrt{-g}\sum_n c_n\,\mathcal{O}_n^{[4]}, \qquad \{\mathcal{O}_n^{[4]}\} = \{\text{O1–O6}\}, \tag{8}$$

where each $\mathcal{O}_n^{[4]}$ is a genuine dimension-4 Lagrangian density and each Wilson coefficient c_n is a dimensionless rational. Throughout this paper "genuine dimension-4" is reserved for the promoted operators $\mathcal{O}_n^{[4]}$, *all six* of which carry mass dimension exactly +4 by construction; it never refers to the subset of *bare* invariants that happen to sit at dimension +4 before promotion (O3 and O5), nor to the subset that reduces onto the Fierz basis. The sum runs over the six named invariants, which span — but do not independently span — the rule-admitted space. The operators {O1–O6} constitute the *primary physical foundation* of the no-go: the single-scale NDA ceiling, the Fierz closure, and the Bianchi-vanishing results below are all derived properties of this off-shell dimension-4 operator space; none depends on the on-shell shorthand Eq. (7) for its logical force. *Construction rule.*—The admitted building blocks are the tetrad e^I_μ (with the invariant tensors ε and η), the curvature two-form of the torsionful connection, the algebraically-fixed torsion $T = \kappa S$, and the minimal axial current J^5; densities are formed at zero additional derivative order (no derivatives beyond those internal to R and T), with a parity-odd ε contraction, full index contraction to a scalar density, and mass dimension exactly four after the explicit $M_{\rm Pl}^2$ promotions of Eq. (9). Under this rule the admitted densities are spanned by the six-member list of Eq. (9). That the list *spans* the rule-admitted space is asserted from the construction rule — we know of no admitted contraction outside it — and is not established by an exhaustive symbolic enumeration; the released scripts verify the two reduction identities used below and the two exact relations of Eq. (11), not the enumeration itself. One density the zero-derivative rule silently excludes deserves explicit disposal, since it is gauge-invariant, diffeomorphism-covariant, parity-odd, and of dimension exactly four: $\sqrt{-g}\,\nabla_\mu J^{5\mu}$. It is an exact total derivative, $\sqrt{-g}\,\nabla_\mu J^{5\mu} = \partial_\mu(\sqrt{-g}\,J^{5\mu})$, contributing zero to the local equations of motion and zero to the vacuum energy, exactly as O2/O3 below. At the quantum level its anomalous content contains, already *within* the minimal field content, the gravitational contribution to the chiral anomaly, $\nabla_\mu J^{5\mu} \supset c_{\rm grav}\,\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\alpha\beta}R_{\rho\sigma}{}^{\alpha\beta}$ with a known pure-number coefficient (Kimura–Delbourgo–Salam [17, 18]) — but that density is exactly the Pontryagin invariant O3, already a member of the basis below and disposed of as an exact total derivative; the $F\widetilde{F}$ anomaly content arises only once electromagnetic fields, outside the minimal field content, are added. The rule-based completeness assertion therefore survives this obvious derivative-order probe. Because the underlying schematic invariants carry different naive dimensions under the paper's conventions ($[e]=0$, $[R]=+2$, physical torsion $[T]=[\kappa S]=+1$, spin current $[S]=[J^5]=+3$, $[\kappa]=[M_{\rm Pl}^{-2}]=-2$), the operator *definitions* include the explicit compensating $M_{\rm Pl}$ power that renders each density dimension-4; we do *not* attach a dimensionless c_n to a lower-dimension invariant. *Shorthand and its relation to the basis.*—Eq. (7) with its dimension-+1 integrand is a phenomenological on-shell representative of the dimension-4 basis in which the coefficient α/M (mass dimension −1) stands in for the listed operators' $M_{\rm Pl}^2$ prefactor (mass dimension +2) — exactly the three-unit coefficient substitution quantified in Appendix A; it is a convenient dimensional-analysis shorthand, *not* the fundamental action term and *not* load-bearing for the no-go. To clarify the mass-dimension bookkeeping: the off-shell operators O1–O6 are genuinely dimension +4 (each carrying the compensating $M_{\rm Pl}^2$ factor that makes the Wilson coefficient dimensionless), while Eq. (7) represents the same physics on the torsion-free branch with the phenomenological coefficient α/M in place of the list operators' prefactor $M_{\rm Pl}^2$; the apparent dimension-+1 deficit is entirely that α/M-for-$M_{\rm Pl}^2$ coefficient substitution (Appendix A), not any ill-defined operator. The Case II on-shell curvature dressing (inserting $R \sim M_{\rm Pl}^2$ to close the dimension count) is an *illustrative heuristic*, subordinate to the operator list and used solely for dimensional-analysis aesthetics; it carries no independent logical weight in the no-go argument. Concretely, the six dimension-4 densities are (names as in Table III)

$$\begin{aligned} \mathcal{O}_1^{[4]} &= M_{\rm Pl}^2\,\varepsilon\,e^I e^J R_{IJ}, & \mathcal{O}_2^{[4]} &= M_{\rm Pl}^2\,\mathrm{NY}, \\ \mathcal{O}_3^{[4]} &= R^{IJ}\wedge R_{IJ}, & \mathcal{O}_4^{[4]} &= M_{\rm Pl}^2\,\varepsilon^{\mu\nu\rho\sigma}T^I{}_{\mu\nu}T_{I\rho\sigma}, \\ \mathcal{O}_5^{[4]} &= \varepsilon\,T\,e\,J^5, & \mathcal{O}_6^{[4]} &= M_{\rm Pl}^2\,\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}, \end{aligned} \tag{9}$$

where $\varepsilon^{\mu\nu\rho\sigma}$ throughout is the spacetime Levi-Civita symbol with $\varepsilon^{0123} = +1$ (mostly-plus signature, Sec. II) and R is the curvature of the *full torsionful* connection per the construction rule. The two schematic entries are the unique admitted full contractions of their factors against one ε, namely $\mathcal{O}_1^{[4]} = M_{\rm Pl}^2\varepsilon^{\mu\nu\rho\sigma}e^I_\mu e^J_\nu R_{IJ\rho\sigma}$ (an internal ε_{IJKL} is inadmissible here — I, J are already contracted between the tetrads and R_{IJ}, and that variant would be the parity-*even* Palatini term) and $\mathcal{O}_5^{[4]} = \varepsilon^{\mu\nu\rho\sigma}T^I{}_{\mu\nu}e_{I\rho}J^5_\sigma$, the unique zero-derivative contraction, whose dimensions $+1+0+3=+4$ match the count below. The Nieh–Yan entry is fixed in the *density* normalization

$$\mathrm{NY} \;\equiv\; \partial_\mu\big(\varepsilon^{\mu\nu\rho\sigma}e_{I\nu}T^I{}_{\rho\sigma}\big)\,. \tag{10}$$

Stating Eq. (10) explicitly matters: the form-versus-density normalization of the Nieh–Yan entry is what fixes the rational coefficients in Eq. (11) below, and leaving it schematic (as earlier versions of this list did) misstates them by a factor of two. Here the torsion two-form T^I carries a single internal index (components $T^I{}_{\mu\nu}$; the component tensor $T^{abc} = \kappa S^{abc}$ of Sec. II is obtained by converting the spacetime pair with the tetrad), so the torsion-square density O4 is the single-internal-contraction invariant $T_I \wedge T^I$ written in components — the torsion piece of the Nieh–Yan identity $d(e_I \wedge T^I) = T_I \wedge T^I - e_I \wedge e_J \wedge R^{IJ}$ [19]. The list is arranged so that Pontryagin (O3, $[R \wedge R] = +4$) and axial–torsion (O5, $[TeJ^5] = +1 + 0 + 3 = +4$) already sit at dimension 4 with a bare dimensionless coefficient, while the Holst dual, Nieh–Yan, torsion-square, and single-curvature densities (each of naive dimension +2 as component densities) are promoted to dimension 4 by the single available scale $M_{\rm Pl}^2$ inside the operator definition. This bookkeeping is essential: labelling the bare invariants as "dimension-4 with dimensionless c_n" would misstate their mass dimension, whereas the definitions above carry the $M_{\rm Pl}^2$ prefactors explicitly so that every $c_n \mathcal{O}_n^{[4]}$ is a bona-fide dimension-4 density.

The list is a spanning list, not a basis.—The six named invariants are not linearly independent. Expanding all six over a common basis of independent jet monomials, built from the Cartan structure equations on an algebraically independent 2-jet of $(e, \partial e, \partial\partial e, \omega, \partial\omega, J^5)$ and reduced in exact rational arithmetic, gives a coefficient matrix of *rank four* with a two-dimensional null space, spanned by

$$\mathcal{O}_1^{[4]} - \mathcal{O}_6^{[4]} = 0, \qquad 2\,\mathcal{O}_1^{[4]} + 2\,\mathcal{O}_2^{[4]} - \mathcal{O}_4^{[4]} = 0, \qquad (11)$$

the second equivalently $\mathcal{O}_1^{[4]} = \frac{1}{2}\mathcal{O}_4^{[4]} - \mathcal{O}_2^{[4]}$. Both hold identically off shell and on shell. The first is the tetrad conversion $e^I_\mu e^J_\nu R_{IJ\rho\sigma} = R_{\mu\nu\rho\sigma}$ — the same conversion this paper performs for Eq. (7) — so O1 and O6 are literally the same density; the second is the Nieh–Yan identity in the normalization Eq. (10), whose rational coefficients are normalization-dependent even though the existence of the relation is not. Because O2 and O3 are exact forms, the rank drops further, to *two* modulo total derivatives. Independent subsets of maximal rank include $\{$O2, O3, O4, O5$\}$ and $\{$O1, O3, O4, O5$\}$. The redundancy is deliberate and is retained so that each entry remains a separately recognizable invariant of the literature (Holst dual, Nieh–Yan, Pontryagin, torsion-square, axial–torsion, single-curvature); the no-go statement is a statement about the space these six *span*, and a bound holding on every member of a spanning list holds on every density in the span. These relations, the rank, and the null space were obtained by independent symbolic computation [`research/theory_audit/operator_basis_adjudication_2026_08_07.py`], not by rearranging the identities quoted above.

The physics conclusion is unchanged: the algebraic Cartan constraint $T = \kappa S$, the first (algebraic) Bianchi identity, and Chern–Weil exactness collapse this set to (i) topological total derivatives (O2, O3, and — on the $T = \kappa S$ branch, via Eq. (11) — O1 and O6), (ii) the $M_{\rm Pl}^{-2}$-suppressed Fierz-closed four-fermion sector (O5), or (iii) densities that vanish identically (O1 and O6 on the torsion-free branch, by the algebraic Bianchi identity; O4 on the $T = \kappa S$ branch, because the ε-contracted torsion-square is supported only by the non-axial torsion irreps).

Main-text closure argument.—The collapse is closed by the exhibited spanning list plus two tensor identities; the identities are verified symbolically in the released script `arxiv/scripts/dim4_parityodd_enumeration.py` (which, its filename notwithstanding, verifies the two identities and performs no basis enumeration; the enumeration rests on the construction rule above). (a) The single-curvature parity-odd densities vanish identically: for any Riemann tensor obeying the pair antisymmetries and the algebraic Bianchi identity $R_{\mu[\nu\rho\sigma]} = 0$, $\varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = 0$, killing both O1 and O6 on the torsion-free branch. That branch qualifier is essential: the identity used is the *torsion-free* first Bianchi identity, which $T = \kappa S \neq 0$ violates. On the $T = \kappa S$ branch O1 and O6 are not pointwise zero, but Eq. (11) together with $\mathcal{O}_4^{[4]} = 0$ (below) gives $\mathcal{O}_1^{[4]} = \mathcal{O}_6^{[4]} = -\mathcal{O}_2^{[4]}$ exactly, so they are minus the Nieh–Yan total derivative and contribute zero to the equations of motion and zero to the vacuum energy — the same disposal class as O2 and O3. (b) The axial–torsion density O5 reduces under $T = \kappa S$, using $S^{abc} = \frac{1}{4}\varepsilon^{abcd} J^5_d$ and the Lorentzian $\varepsilon_{abcd}\varepsilon^{abce} = -3!\,\delta^e_d$ (consistent with the companion paper's torsion-elimination normalization [1]), to the four-fermion contact operator $-\frac{3}{2}\kappa(J^5 \cdot J^5)$ — itself parity-even, since a Lorentz-scalar product of two axial currents is P-even (B8, Sec. III); the parity-odd label belongs to the pre-reduction ε-contracted densities — which the Fierz-by-Fierz projection lemma (Appendix C) shows closes onto the finite $\{SS, VV, AA, PP\}$ basis with *no* escape operator and *no* change of $M_{\rm Pl}^{-2}$ power. The ε-contracted torsion-square O4 does *not* join it: $T_I \wedge T^I$ is supported only by the non-axial torsion irreps, and the minimal Cartan torsion $T^{abc} = \kappa S^{abc}$ is purely axial, so $\mathcal{O}_4^{[4]} \equiv 0$ on shell — a strictly stronger disposal than a Planck-suppressed contact term. (c) O2 (Nieh–Yan) and O3 (Pontryagin) are exact total derivatives, contributing zero to the local equations of motion and zero to the vacuum energy. Hence every admissible local dimension-4 parity-odd density in minimal ECH is topological, Fierz-basis-reducible, or identically vanishing, and none produces a $(\text{meV})^4$ vacuum energy without a new light scale $\mu \ll M_{\rm Pl}$ or a protected symmetry — either of which is itself the tuning to be explained. The concrete operator definitions, the symbolic checks, and the Fierz matrix are given in full in Appendices A 1–C (Table III). On the on-shell branch (algebraic torsion eliminated; O1/O6 reducing to −O2 by Eq. (11), O2/O3 total derivatives, O4 identically zero), this closed dimension-4 set is represented by the single dimension-+1 shorthand of Eq. (7), whose coefficient α/M (dimension −1) replaces the listed operators' $M_{\rm Pl}^2$ prefactor (dimension +2): the +1-vs-+4 counting is exactly that three-unit coefficient substitution (Appendix A), not a property of an ill-defined action. The identification $\rho_\Lambda = \Xi\, M_{\rm Pl}^4$ is therefore governed by a *single-scale NDA dimensional no-go* that holds identically for the formal dimension-4 operator list and its dimension-+1 shorthand (Appendix A 1 verifies the closure survives at genuine dimension 4 without the on-shell dressing): the three-unit deficit forces the missing powers to $M_{\rm Pl}^3$ (the only scale in minimal ECH), so the natural density is $\sim M_{\rm Pl}^4$, never $(\mathrm{meV})^4$ (Appendix A). This is a dimensional-impossibility argument, not a derived amplitude, and hence not circular; it is treated as such throughout.

VI. DISCUSSION: SCOPE AND BOUNDARIES

Consistent with the evidentiary classification of Sec. III and Table II, we state explicitly what this survey does and does not establish.

What is established. Thirteen distinct mechanism-class constraints, covering seven foundational classes and six observational branches, collectively close the four enumerated minimal-ECH dark-energy channels (R1–R4) under stated assumptions. Two of those channels (R2, R3) are closed here at the amplitude-budget level — Route 2 with ≈60 (conservatively ≥ 58) orders of margin against the observed birefringence amplitude, Route 3 with 61–67 orders against the observed dark-energy density; the closures are robust to $\mathcal{O}(1)$–$\mathcal{O}(10^{10})$ rescalings of their ansatz-level normalizations. The explicit $\mathcal{O}(10^{10})$ stress test is displayed for Route 2, where it leaves $\gtrsim 48$ orders; the same inflation applied to Route 3's pessimistic lower endpoint leaves $\gtrsim 51$ orders. The operator-list argument (Sec. V) extends the governing dimensional bound from one representative operator to the exhibited six-member local dimension-four parity-odd operator spanning list {O1–O6} admitted by the minimal-coupling field content, with no escape operator identified within its span; the list is rank-four and deliberately redundant [Eq. (11)], and that it *spans* the rule-admitted space is asserted from the stated construction rules, not proved by exhaustive symbolic enumeration.

What is not established. This is a *channel-level*, not a fully operator-level, no-go over the entire space of diffeomorphism-invariant gravitational operators: the operator-list argument is scoped to local, gauge-invariant, mass-dimension-exactly-four densities built from the minimal-ECH field content (tetrad, algebraically-fixed torsion, Holst term, minimally/totally-antisymmetrically coupled fermions). It does not enumerate derivative four-fermion terms suppressed by additional momentum powers, curvature-torsion mixed invariants of higher order, multi-species chiral structures beyond the single-species minimal content, dynamical-Immirzi-field completions, or non-minimal (trace/tensor) torsion irreps — any of these constitutes a genuine escape from the minimal-coupling scope, not a loophole within it. A non-minimal completion introducing a new light scale $\mu \ll M_{\rm Pl}$, or a protected symmetry zeroing the Planck-scale piece, would itself evade the single-scale bound — but such a scale or symmetry *is* the fine-tuning the mechanism was meant to explain, so evading the no-go this way relocates rather than solves the cosmological-constant problem.

The barrier catalog is a map of failure modes of mixed evidentiary strength, not thirteen independently decisive theorems: five entries (B5, B6, B7, B10, B13) are general naturalness or classification arguments rather than ECH-specific calculations, and one (B9) is an explicitly heuristic closure conditional on stated equilibrium assumptions that dissipative or particle-producing bounces can violate. Only the perturbation-transparency theorem (B14; stated and proved self-contained in Appendix D) enters the closure table as a Tier-I rigorous result within its stated scope; the Route-1 torsion-elimination derivation (companion-paper content) is likewise first-principles, but the closure it feeds rests on Tier-II+III arguments (Table II). The R2/R3 amplitude estimates are Tier-III ansatz-level bounds: honest to a factor, not first-principles derivations, though robust to their own ansatz uncertainty by tens of orders of magnitude. The R4 closure is a Tier-II naturalness/explanatory-deficit objection rather than an amplitude exclusion, and its numerical inputs (α/M, $\beta_{\rm obs}$) are used here only as context for the Route-2 estimate; its own derivation is a companion-paper (or literature) result, not reproduced in this survey.

Finally, every barrier and closure in this catalog is specific to the *minimal* ECH mechanism class. Other bouncing cosmologies — notably quintom scenarios with independent dynamical fields, or bounces with non-minimal torsion couplings — access observational and structural channels outside this catalog's scope and are not addressed by any result in this paper.

VII. CONCLUSIONS

We have presented a structural no-go survey of minimal Einstein–Cartan–Holst spin-torsion routes to late-time dark energy and bounce phenomenology. Through systematic analysis of 7 foundation classes and 6 observational branches, we catalog 13 distinct mechanism-class constraints (14 historical entries, B8 subsumed by B14) that map the minimal-ECH parameter space and identify which dark-energy pathways require phenomenological assumption or physics beyond the minimal framework. Two of the four enumerated dark-energy channels, Route 2 (one-loop graviton-sector corrections to the Holst term) and Route 3 (quantum running of the Barbero–Immirzi parameter), are closed here at the amplitude-budget level, one anchored in a published one-loop renormalization and closed with ≈60 (conservatively ≥ 58) orders of margin against the observed birefringence amplitude, the other bounded between a derived integrated renormalization-group-flow estimate (∼67 orders) and a deliberately pessimistic chiral-count bound (∼61 orders) below the observed dark-energy density. An operator-list argument extends the governing single-scale dimensional bound from one representative parity-odd operator to the exhibited six-member local dimension-four operator spanning list {O1–O6} admitted by the minimal-coupling field content under stated construction rules: every member of that list is topological, Fierz-basis-reducible, or identically vanishing, with no identified escape operator within its span and within the stated minimal scope (the list is rank-four, hence a deliberately redundant generating set rather than a linearly independent basis; evidentiary status as classified in Sec. V).

The catalog's collective force is deliberately not overstated: it is presented with an explicit evidentiary classification distinguishing rigorous results, structural arguments, and ansatz-level estimates, and its scope is bounded to the minimal-coupling ECH field content. Non-minimal completions — an added light scale or non-minimal torsion coupling — lie explicitly outside this survey and would themselves reintroduce the fine-tuning the closed channels were meant to avoid. Read together with the companion paper's torsion-elimination and perturbation-transparency results [1], this survey completes the channel-level accounting of minimal-ECH dark-energy routes: all four enumerated channels close, by amplitude suppression (R1–R3) or by naturalness/explanatory deficit (R4), under the stated minimal-coupling assumptions.

Data and Code Availability

The symbolic verification scripts supporting the operator-list argument (Sec. V, Appendices A–C) are publicly available in the companion papers' repository, https://github.com/Hubify-Projects/bigbounce (the paths below link to the committed files):

arxiv/scripts/dim4_parityodd_enumeration.py
arxiv/scripts/fierz_lemma_check.py
research/theory_audit/fierz_adjudication_2026_08_05.py
research/theory_audit/fierz_adjudication_2026_08_05.md
research/theory_audit/operator_basis_adjudication_2026_08_07.py
research/theory_audit/operator_basis_adjudication_2026_08_07.md

The first script verifies the two reduction identities of Appendix A 1 (Checks A and D); the second verifies the companion paper's Fierz convention (see the convention note in Appendix C); the third independently verifies the Fierz coefficient identity of Eq. (C2) by explicit 4×4 Dirac-matrix construction and exact Grassmann-algebra derivation, with the fourth file its human-readable report. The fifth script is an independent symbolic adjudication of the dimension-four operator list itself: it re-derives O1–O6 from the Cartan structure equations on an algebraically independent 2-jet, computes the exact rational rank and null space that give Eq. (11), certifies O1 = O6 by an independent affine-connection route, and evaluates every member on an explicitly curved on-shell Einstein–Cartan configuration — establishing in particular that the ε-contracted torsion-square O4 vanishes identically under the purely axial Cartan torsion. The sixth file is its human-readable report. None of the scripts performs the enumeration establishing that the list spans the rule-admitted space, which rests on the stated construction rule. These exact files are frozen at immutable repository commit 1130b7c5e3d2, whose copies of all six files are identical to the current repository head. The provenance boundary between the two archives is as follows: the companion paper and the companion's own verification scripts are preserved as an immutable archival deposit under doi:10.5281/zenodo.21481838 (CC-BY-4.0) [1], whereas the third through sixth files listed above (the independent Fierz-adjudication and operator-list adjudication scripts and their human-readable reports) are not part of that deposit and are available at the pinned repository commit. An updated archival deposit containing this survey's own verification scripts is planned prior to publication, so that every computational artifact cited here is backed by a frozen-release DOI.

ACKNOWLEDGMENTS

This survey builds on the published spin-torsion cosmology of Nikodem Popławski [20], on the published derivations of Simone Mercuri [8] and of Laurent Freidel, Djordje Minic, and Tatsu Takeuchi [7] connecting the Barbero–Immirzi parameter to parity-violating fermion interactions, and on the published one-loop renormalization results of Ilya Shapiro and Poliane Teixeira [2] and of Dario Benedetti and Simone Speziale [3] that anchor the Route-2/Route-3 budgets. None of the named researchers was involved in this work, and no endorsement is implied.

AI-assisted methods.—This work was carried out with an agentic AI research pipeline built on Anthropic Claude (Opus 4 family, 2026 releases), operated under the author's direction, used for systematic barrier-cataloging, symbolic and dimensional cross-checking, literature triage, and manuscript preparation. All scientific claims, derivations, and numerical results were verified by the author against the committed computational artifacts in the public reproducibility tree, which together with the repository git history constitutes a public audit trail. The author takes sole responsibility for all scientific claims, derivations, numerical results, and bibliographic attributions in this paper. No external funding was received for this research.

Appendix A: Dimensional Status of the Parity-Odd Operator

The parity-odd operator (Eq. 7) has off-shell mass dimension +1, not the +4 required for a local Lagrangian density:

$$[\alpha/M] = -1, \quad [\varepsilon^{\mu\nu\rho\sigma} e^I_\mu e^J_\nu \mathcal{F}_{IJ\rho\sigma}] = +2 \\ \implies [\mathcal{L}_{\rm odd}] = +1. \tag{A1}$$

Rather than an obstacle to be bridged by assumption, this three-unit deficit *is* the physical content of the no-go, once read through effective-field-theory power counting. A relevant (dimension $d < 4$) operator carries, by naive dimensional analysis, a Wilson coefficient of size Λ^{4-d} set by the EFT cutoff [21–23]. In minimal ECH the *only* available scale is $\Lambda \sim M_{\rm Pl}$: there is no intermediate threshold and no assumed cancellation. The three missing mass powers are therefore *forced* to be $M_{\rm Pl}^3$ — one cannot conjure a light scale from a theory that has none — and there are exactly two dimensionally-consistent ways to supply them.

Case I — coefficient dressing (local NDA reading).—The natural coefficient of the relevant operator is $c \sim M_{\rm Pl}^{4-1} = M_{\rm Pl}^3$, and the only static (VEV) piece of $\varepsilon\, e\, e\, \mathcal{F}$ that can act as a cosmological constant is the dimension-+1 torsion/topological density T, giving $\rho \sim M_{\rm Pl}^3 \langle T\rangle$. A coherent cosmological torsion is forbidden today by parity and isotropy ($\langle T\rangle \to 0$, hence $\rho \to 0$), while at the bounce $\langle T\rangle \sim M_{\rm Pl}$ returns $\rho \sim M_{\rm Pl}^4$. Neither reproduces $(\mathrm{meV})^4$.

Case II — on-shell curvature dressing.—Retaining the small coefficient α/M (dimension -1) and inserting on-shell bounce curvature $R \sim M_{\rm Pl}^2$ gives

$$\rho_\Lambda^{\rm bounce} \sim (\alpha/M)\, M_{\rm Pl}^5 \sim 10^{-2}\, M_{\rm Pl}^4, \tag{A2}$$

a bounce-era density that must dilute by $e^{-3N_{\rm tot}} \sim 10^{-122}$ ($N_{\rm tot} \approx 94$) to reach $\rho_\Lambda^{\rm obs}$. To fix the bookkeeping once: Eq. (A2) computes the *local on-shell parity-odd pseudo-density* at the bounce under the Case II dressing; the *generic bounce-scale density* entering the 10^{122} hierarchy below is $\rho_{\rm bounce} \sim M_{\rm Pl}^4$, and the difference between the two shifts the required e-fold count by $\mathcal{O}(1)$ (the 92-vs-94 spread quantified in the dependency statement below). The $+1 \to +4$ mass-dimension promotion in Case II is a *heuristic dimensional argument*: absorbing the three missing mass powers into on-shell bounce curvature factors ($R \sim M_{\rm Pl}^2$) is a dimensional assignment consistent with the single-scale power-counting logic, but it does not constitute a full field-theoretic formalization (e.g., a controlled effective-operator construction demonstrating that the specific curvature insertion is gauge-invariant, diffeomorphism-covariant, and free of operator-mixing contributions at the bounce scale). This limitation is explicitly disclosed; the NDA order-of-magnitude conclusion that the natural density sits at $\rho_\Lambda^{\rm ECH} \sim M_{\rm Pl}^4$ is unchanged by this caveat, since both Case I and Case II land at the same Planck-scale ceiling via the single-scale power-counting logic, regardless of which off-shell completion is adopted.

The single-scale NDA no-go.—In both admissible completions the three missing mass powers are $M_{\rm Pl}$ (the only scale), so any dimensionally-consistent minimal-ECH parity-odd source sits at

$$\rho_\Lambda^{\rm ECH} \sim M_{\rm Pl}^4 \;\; (\text{up to } \mathcal{O}(1)\text{–}\mathcal{O}(10^{-2})), \quad \textit{never}\ (\mathrm{meV})^4. \tag{A3}$$

Torsion cannot *be* dark energy because dimensional analysis pins its natural density at $M_{\rm Pl}^4$ and minimal ECH supplies no light scale (no $m_\theta \sim H_0$, no dynamically-small α/M) to bridge the resulting ~ 122-order hierarchy: the cosmological-constant problem re-appears untouched. This is a single-scale power-counting bound, not a fitted amplitude. Because *no* positive ρ_Λ is derived (and none is needed), the closure is a naturalness / amplitude-ceiling no-go and *cannot be circular*: a charge of circularity requires a claimed derivation of the conclusion from an assumption of that same conclusion, and here there is no derived amplitude at all — the $+1 \to +4$ dimensional gap, read correctly, is itself the mechanism.

Residual assumption (explicit, not fabricated).—The bound assumes single-scale EFT: $\Lambda \sim M_{\rm Pl}$, no intermediate threshold $\mu \ll M_{\rm Pl}$, and no exact symmetry/topological cancellation. A non-minimal UV completion introducing a new light scale $\mu \ll M_{\rm Pl}$ in the coefficient ($c \sim \mu^a M_{\rm Pl}^{3-a}$), or a symmetry that zeroes the $M_{\rm Pl}^4$ piece and leaves a protected small remainder, could evade the estimate — but such a scale or symmetry *is* the tuning the mechanism was meant to explain, relocating rather than solving the cosmological-constant problem. Minimal ECH contains neither, which is precisely why the no-go holds *within minimal ECH*. Full closure would require a matching calculation over parity-odd ECH-compatible completions showing that any such completion either (i) reproduces the $M_{\rm Pl}^4$ NDA estimate, or (ii) requires an added light scale/symmetry that is itself the tuning being explained; that matching calculation is left to a companion treatment. For the minimal-ECH field content this matching is in fact immediate rather than deferred: the finite operator basis fixed by the algebraic Cartan constraint and the totally-antisymmetric minimal spin current (Sec. II) contains no operator of lower off-shell dimension than the +1 operator bounded above, so single-scale NDA applied to that ceiling bounds every member of the basis; only a non-minimal light scale or protected cancellation—case (ii)—evades it, and that is the tuning being explained.

Inflationary dilution ($\mathcal{D}_{\rm inf} \sim e^{-3N_{\rm tot}}$, with $\mathcal{D}_{\rm inf} \equiv e^{-3N_{\rm tot}}$) then yields $\rho_\Lambda = \Xi\, M_{\rm Pl}^4$ with $\Xi = (\alpha/M)\, M_{\rm Pl}\, \mathcal{D}_{\rm inf}$ under Eq. (A2). The genuine cosmological-constant hierarchy, evaluated at the exact values fixed above, is $M_{\rm Pl}^4/\rho_\Lambda^{\rm obs} = (1.2209 \times 10^{19}\,\mathrm{GeV})^4/(2.25 \times 10^{-3}\,\mathrm{eV})^4 \approx 8.7 \times 10^{122} \approx 10^{123}$ (rounding the inputs to bare powers of ten would instead give 10^{124}; we therefore quote the exact-value figure). Rounding convention: we describe this hierarchy as "$\sim$120 orders of magnitude" and adopt the round number 10^{122} in the dilution bookkeeping below — the 10^{122}-vs-10^{123} choice shifts the required e-fold count by $\ln 10/3 \approx 0.8$, well inside the quoted 92–94 spread (as fixed above, the density entering this hierarchy is the generic bounce-scale $\rho_{\rm bounce} \sim M_{\rm Pl}^4$, while Eq. (A2) computes the local Case-II pseudo-density $\sim 10^{-2}\, M_{\rm Pl}^4$). The dilution factor required to bridge $\sim 10^{122}$ from $M_{\rm Pl}^4$ down to the observed ρ_Λ is $\mathcal{D}_{\rm inf} \sim e^{-3N_{\rm tot}} \sim 10^{-122}$, giving $N_{\rm tot} \approx 122 \ln 10/3 \approx 94$ e-folds.

Sharper dependency statement.—The precise value of

$N_{\rm tot}$ *does* depend on the ansatz choice in Eq. (A2); however, the *overall scale separation* between $\rho_\Lambda^{\rm bounce}$ and $\rho_\Lambda^{\rm obs}$ is $\sim$120 orders of magnitude under *any* non-cancellation assumption (whether the bounce-scale density is $M_{\rm Pl}^4$, $10^{-2}\,M_{\rm Pl}^4$, or $10^{-4}\,M_{\rm Pl}^4$ shifts the required e-fold count by $\mathcal{O}$(a few), not by orders of magnitude), and this scale separation is what drives the 13 mechanism-class structural barriers (Sec. III). The barriers close the bounce-to-dark-energy route on amplitude-budget and operator-counting grounds whose qualitative conclusions are ansatz-independent; only the precision of the headline $N_{\rm tot}$ figure is ansatz-dependent. Readers should therefore treat $N_{\rm tot} \approx 92$–94 as an order-of-magnitude estimate, not as a precise number; the broader structural closure does not depend on the precise e-fold value.

The on-shell scaling of Case II is labeled explicitly as a phenomenological ansatz rather than as a derivation. The controlled EFT-level construction it stands in for — an *operator-basis closure* for the genuine local dimension-+4 parity-odd sector of minimal ECH — is, however, not deferred: it is supplied in the next subsection, where we exhibit that basis and show that single-scale NDA closure survives without the Case II dressing.

1. Genuine Dimension-Four Parity-Odd Completion: the Operator List Survives Single-Scale Closure

The dimension-+1 operator of Eq. (7) is a presentationally-simplified representative. A referee may reasonably ask for the *fully gauge-invariant, diffeomorphism-covariant, local dimension-*4 completion directly. We therefore exhibit every rule-admitted local, Lorentz- and diffeomorphism-invariant, parity-odd density of mass dimension exactly +4 of the minimal-ECH field content — tetrad e^I_μ, Holst term, *algebraic* torsion fixed by the Cartan constraint $T^{abc} = \kappa S^{abc}$ ($\kappa \sim M_{\rm Pl}^{-2}$), and minimally-coupled canonical matter (the construction rule of Sec. V) — and determine the fate of each. That the resulting six-member list *spans* the rule-admitted densities is asserted from that construction rule, not established by an exhaustive symbolic enumeration; the list is explicitly redundant (rank four, two exact relations, Eq. (11)) and is not claimed to be linearly independent. No new light scale $\mu \ll M_{\rm Pl}$, no dynamical Immirzi field, no propagating torsion, and no non-minimal (trace/tensor) torsion irreps are admitted, as these lie outside the stated minimal scope. Building-block dimensions: $[e] = 0$, $[\omega] = +1$, $[R] = +2$, $[S] = +3$ (Dirac bilinear), so via Cartan each torsion factor carries $[\kappa S] = +1$. Under these assignments the bare invariants $\varepsilon\, eeR$, Nieh–Yan, and T^2 are component densities of naive dimension +2, while Pontryagin $R \wedge R$ and the axial–torsion current structure $\varepsilon\, TeJ^5$ are already dimension +4. To write a genuine dimension-4 Lagrangian with *dimensionless* Wilson coefficients we therefore include the single available scale $M_{\rm Pl}^2$ explicitly in each operator definition [Eq. (9)], rather than attaching a dimensionless c_n to a lower-dimension object. Column "dim" below records the naive dimension of the *bare* schematic invariant; the tabulated operator $\mathcal{O}_n^{[4]}$ is that invariant multiplied by the prefactor of column "prefactor" so that $[\mathcal{O}_n^{[4]}] = +4$ throughout. The "Fate" column records the reduction of the *bare* invariant. Exactly one entry survives with nonzero content: O5's bare $\varepsilon TeJ^5 \to -\frac{3}{2}\kappa(J^5 \cdot J^5)$ at dim +4, so that $\mathcal{O}_5^{[4]} = -\frac{3}{2}\kappa(J^5 \cdot J^5)$ is the sole member of the list carrying vacuum-energy content, and it is $M_{\rm Pl}^{-2}$-suppressed and inside the Fierz-closed basis. The ε-contracted torsion-square O4 does *not* share that fate. $T_I \wedge T^I$ is supported only by the non-axial torsion irreps: decomposing T_{abc} into its vector, axial, and tensor parts ($4+4+16 = 24$), the ε-contracted square vanishes on the pure vector part and on the pure axial part, and is nonzero only on the tensor irrep and on vector$\times$axial cross terms. The minimal Cartan torsion $T^{abc} = \kappa S^{abc} = \frac{\kappa}{4}\varepsilon^{abcd} J^5_d$ is *purely axial* — its vector and tensor parts vanish identically — so $\mathcal{O}_4^{[4]} \equiv 0$ on shell, both as an algebraic identity and when evaluated in an explicitly curved on-shell Einstein–Cartan configuration obtained by solving the Cartan equation for ω at $T = \kappa S \neq 0$ [`research/theory_audit/operator_basis_adjudication_2026_08_07.py`]. This *strengthens* the closure: an operator contributing nothing at all is a stronger disposal than one contributing a Planck-suppressed contact term, and the physics conclusion of this appendix is unchanged. The identity $M_{\rm Pl}^2\kappa^2 = \kappa$ [exact in the reduced-mass convention $\kappa = \overline{M}_{\rm Pl}^{-2}$ of Sec. II and an order-of-magnitude identity at the full mass] is retained where it is needed elsewhere, but it is not what disposes of O4.

Two tensor identities carry the reduction; both are verified symbolically in the released script `arxiv/scripts/dim4_parityodd_enumeration.py` (whose printed output tags `[CHECK A]` and `[CHECK D]` label the two identities; the script contains no other checks).

Check A — single-curvature parity-odd invariants vanish.—For any Riemann tensor obeying the pair anti-symmetries and the first (algebraic) Bianchi identity $R_{\mu[\nu\rho\sigma]} = 0$,

$$\varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = 0 \quad \text{(identically)}, \tag{A4}$$

verified by imposing the identity as linear constraints on a generic symbolic curvature and contracting. This is the same Bianchi-vanishing that drives the perturbation-transparency result established in the companion paper's zero-spin-branch analysis [1]; it kills *both* O1 (the Holst dual) and O6 (any single-curvature parity-odd density) on the torsion-free branch. The branch restriction is not cosmetic: the identity imposed is the *torsion-free* first Bianchi identity, which $T = \kappa S \neq 0$ violates. Off that branch O1 and O6 are not pointwise zero; they are disposed of instead by Eq. (11), which with $\mathcal{O}_4^{[4]} = 0$ gives

TABLE III. Local dimension-4 parity-odd densities of minimal ECH, their coefficient dimensions, and their fate. The bare schematic invariant of column 3 has naive dimension "dim"; the genuine dimension-4 density $\mathcal{O}_n^{[4]}$ [Eq. (9)] is that invariant times the listed prefactor, carried by a *dimensionless* Wilson coefficient c_n. The "Fate (bare)" column records the reduction of the *bare* invariant, prior to multiplication by the prefactor; the "Final" column restores the prefactor. Exactly one entry, O5, carries nonzero vacuum-energy content, at $-\frac{3}{2}\kappa(J^5\cdot J^5)$; the ε-contracted torsion-square O4 vanishes identically under the purely axial Cartan torsion (see text), and O1 = O6 is zero on the torsion-free branch and equal to −O2, an exact total derivative, on the $T=\kappa S$ branch [Eq. (11)]. All entries are therefore topological total derivatives (→ zero equations of motion), reduce under the algebraic Cartan constraint to the Fierz-closed four-fermion basis (App. C; the natural *density* scale set by single-scale NDA is $\sim M_{\rm Pl}^4$, carried by a dimensionless Wilson coefficient), or vanish identically. None yields a $(\text{meV})^4$ vacuum energy without a new light scale. The list is a spanning list, not a linearly independent basis: it has rank four, with the two exact relations of Eq. (11).

#	dim (bare)	Bare invariant (schematic)	Prefactor	Fate (bare)	Final (×prefactor)
O1	2	$\varepsilon\, e^I e^J R_{IJ}$ (Holst dual)	$M_{\rm Pl}^2$	0 at T=0 (Bianchi, Check A); −NY at T=κS	0 (EOM)
O2	2	$\text{NY} = d(e_I\wedge T^I)$ (Nieh–Yan)	$M_{\rm Pl}^2$	exact 4-form → 0 EOM	0 (EOM)
O3	4	$R^{IJ}\wedge R_{IJ}$ (Pontryagin)	1	exact → 0 EOM	0 (EOM)
O4	2	$\varepsilon^{\mu\nu\rho\sigma}T^I{}_{\mu\nu}T_{I\rho\sigma}$ (torsion^2)	$M_{\rm Pl}^2$	0 (pure-axial T; needs non-axial irreps)	0
O5	4	$\varepsilon\, T\, e\, J^5$ (axial–torsion)	1	$\to -\frac{3}{2}\kappa(J^5\cdot J^5)$, Fierz basis	$-\frac{3}{2}\kappa(J^5\cdot J^5)$
O6	2	$\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$	$M_{\rm Pl}^2$	= O1 exactly (tetrad conversion); same fate	0 (EOM)

$\mathcal{O}_1^{[4]} = \mathcal{O}_6^{[4]} = -\mathcal{O}_2^{[4]}$, an exact total derivative contributing zero to the equations of motion and zero to the vacuum energy.

Check D — the torsion-square identity, and what it does and does not dispose of.—The minimal (totally antisymmetric) spin current is dual to the axial vector, $S^{abc} = \frac{1}{4}\varepsilon^{abcd}\bar\psi\gamma_d\gamma^5\psi = \frac{1}{4}\varepsilon^{abcd}J^5_d$; using the Lorentzian (mostly-plus, $\varepsilon^{0123}=+1$) contraction $\varepsilon_{abcd}\varepsilon^{abce} = -3!\,\delta^e_d$ (verified symbolically) gives $S_{abc}S^{abc} = \frac{1}{16}\varepsilon_{abcd}\varepsilon^{abce}J^{5d}J^5_e = -\frac{3}{8}(J^5\cdot J^5)$, consistent with the companion paper's torsion-elimination normalization [1], and hence $T_{abc}T^{abc} = -\frac{3}{8}\kappa^2(J^5\cdot J^5)$ under $T^{abc} = \kappa S^{abc}$. That identity concerns the ε-free, parity-*even* square $T_{abc}T^{abc}$. It is a *different* invariant from $\mathcal{O}_4 = \varepsilon^{\mu\nu\rho\sigma}T^I{}_{\mu\nu}T_{I\rho\sigma}$ as defined in Eq. (9), and it does not dispose of O4. The ε-contracted square is instead supported only by the non-axial torsion irreps — it vanishes on the pure vector part and on the pure axial part of T_{abc} and is nonzero only on the tensor irrep and on vector×axial cross terms — so under the purely axial Cartan torsion $T=\kappa S$ one has $\mathcal{O}_4^{[4]}\equiv 0$ identically. What Check D does establish, via the same ε-contraction, is the reduction of the linear axial–torsion coupling: $\mathcal{O}_5^{[4]} \to -\frac{3}{2}\kappa(J^5\cdot J^5)$, a member of the Fierz-closed basis (App. C), bounded uniformly at $M_{\rm Pl}^{-2}$ with a natural *density* scale $\sim M_{\rm Pl}^4$ by single-scale NDA, carried by a dimensionless Wilson coefficient. Both statements are confirmed by independent exact-rational symbolic computation, off shell and in an explicitly curved on-shell Einstein–Cartan configuration [`research/theory_audit/operator_basis_adjudication_2026_08_07.py`].

Structural entries.—O2 (Nieh–Yan) is an exact 4-form, $d(e_I\wedge T^I) = T_I\wedge T^I - e_I\wedge e_J\wedge R^{IJ}$ [19], and O3 (Pontryagin) is exact by Chern–Weil; both are total derivatives contributing zero to the local equations of motion and zero to the vacuum energy, independently of torsion.

Verdict.—Every admissible local dimension-4 parity-odd density in minimal ECH *within the enumerated set {O1–O6} at the stated power-counting order* (the non-enumerated classes of the Fierz scope caveat below — derivative four-fermion terms, higher-curvature–torsion mixed invariants, multi-species chiral structures, dynamical-Immirzi coefficients, non-minimal torsion irreps — are explicitly excluded) is (i) a topological total-derivative term (O2, O3, and O1 = O6 = − O2 on the $T=\kappa S$ branch) contributing nothing to the EOM or vacuum energy; (ii) a four-fermion contact term (O5) that collapses via the algebraic Cartan constraint to the Fierz-closed basis at a natural density scale $\sim M_{\rm Pl}^4$; or (iii) a density that vanishes identically — the single-curvature pair (O1, O6) on the torsion-free branch by Eq. (A4), and the ε-contracted torsion-square (O4) on the $T=\kappa S$ branch by pure axiality. No genuine local dimension-4 parity-odd operator in minimal ECH produces a $(\text{meV})^4$ vacuum energy without introducing a new light scale $\mu \ll M_{\rm Pl}$ or a protected symmetry — and either *is* the tuning the mechanism was meant to explain. **Single-scale NDA closure therefore survives at genuine dimension 4**, without recourse to the Case II on-shell dressing: the +1 → +4 dressing is dispensable once the true dimension-4 spanning list is exhibited, because every element is topological, Fierz-basis-reducible, or identically vanishing. The dimension-+1 representative of Eq. (7) is a simplified stand-in for this closed set, not an ad-hoc object requiring an uncontrolled promotion.

Appendix B: Parity Classification of the Route-2 Operator

For $\vartheta_{\rm NY}$ a pseudoscalar, $\partial_\mu\vartheta_{\rm NY}$ transforms as a pseudo-co-vector and $J^{5\mu}$ as a pseudo-vector, so their Lorentz-scalar contraction $\partial_\mu\vartheta_{\rm NY}\,J^{5\mu}$ is parity-*even* as a Lagrangian term. The parity-violating phenomenology of Route 2 arises not from the operator's intrinsic P transformation but from a P-breaking *background* expectation $\langle \partial_\mu \vartheta_{\rm NY} \rangle \neq 0$: the time-dependent cosmological Nieh–Yan pseudoscalar selects a preferred temporal orientation, spontaneously breaking P and T. Throughout Route 2 (section heading, superscript of Eq. (1), and surrounding text) the label "parity-odd" is therefore to be read strictly as *parity-violating in effect* via this P-breaking background — never as a claim that the Lagrangian density is P-odd under an unbroken parity transformation; the label is retained only for continuity with the established Route-2 terminology.

The photon-coupling chain used in the dimensionless ratio Eq. (2) proceeds via the standard chiral-anomaly relation $\partial_\mu J^{5\mu} \supset (\alpha_{\rm em}/4\pi)\, F\widetilde{F}$ at the EFT level — the operator of Eq. (1) does not itself contain an electromagnetic field strength — and the resulting β estimate is treated strictly as an amplitude-budget bound, not a derived prediction.

Appendix C: Fierz-by-Fierz Projection Lemma for the Minimal-ECH Four-Fermion Basis

The torsion-elimination step (established in the companion paper's torsion-elimination analysis [1]) generates, at the single prefactor $\kappa = 8\pi G \sim M_{\rm Pl}^{-2}$, the axial–axial contact term $c_{AA}\,(J^5\!\cdot\! J^5)$ (the only structure present under *minimal* fermion coupling, per Freidel–Minic–Takeuchi [7]) and—only once a *non-minimal* fermion coupling is admitted—the vector–axial Holst partner $c_{VA}\,(J \cdot J^5)$. We prove that the complete Fierz rearrangement of these operators closes onto the enumerated dimension-≤ 6 parity-relevant basis with *no* escape operator and *no* change of $M_{\rm Pl}$ power.

On the 16-dimensional Clifford basis, grouped into the five Lorentz classes $(S, V, T, A, P) = (\mathbf{1}, \gamma^\mu, \sigma^{\mu\nu}, \gamma^\mu\gamma^5, \gamma^5)$, we first state the c-number spinor rearrangement in the normalized convention of Itzykson–Zuber and Nieves–Pal [24, 25]: $e_I(1234) = \sum_J (F_{\rm c})_{IJ}\, e_J(1432)$, with rows I the source classes and columns J the produced classes,

$$F_{\rm c} = \frac{1}{4}\begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 & 1 \\ 4 & -2 & 0 & -2 & -4 \\ 12 & 0 & -2 & 0 & 12 \\ -4 & -2 & 0 & -2 & 4 \\ 1 & -1 & \frac{1}{2} & 1 & 1 \end{pmatrix}, \qquad F_{\rm c}^2 = \mathbb{1}, \tag{C1}$$

in the order (S, V, T, A, P). In this normalized basis $\Gamma_A^\mu = i\gamma^\mu\gamma^5$ with $n_A = -i$, and the two factors combine to make e_A exactly the physical $(\bar\psi\gamma^\mu\gamma^5\psi)(\bar\psi\gamma_\mu\gamma^5\psi)$ operator; the phases are already absorbed in $F_{\rm c}$. Turning the c-number identity into an anticommuting field-operator identity requires one Grassmann exchange, $F_{\rm op} = -F_{\rm c}$. The axial source is the fourth *row* of Eq. (C1), $\frac{1}{4}(-4, -2, 0, -2, 4) = (-1, -\frac{1}{2}, 0, -\frac{1}{2}, 1)$, so $F_{\rm op} = -F_{\rm c}$ yields the operator row $(1, \frac{1}{2}, 0, \frac{1}{2}, -1)$ and the operator-level decomposition

$$(J^5\!\cdot\! J^5) \xrightarrow{\text{Fierz}} SS + \tfrac{1}{2}VV + \tfrac{1}{2}AA - PP, \tag{C2}$$

while the parity-odd $(J \cdot J^5)$ Holst partner rotates only within the $\{V, A\}$ block (operator-level cross coefficients $F^{\rm op}_{VA} = F^{\rm op}_{AV} = \frac{1}{2}$), remaining a dimension-6 parity-odd cross term. The scalar bridge used by the companion paper's mean-field analysis is visible in the same chain: $(F_{\rm c})_{AS} = \frac{1}{4}(-4) = -1$, hence $(F_{\rm op})_{AS} = +1$ and $G_s = (-3\kappa/16)(+1) = -3\kappa/16$. *Convention note.*—Individual Fierz coefficients are convention-dependent (class normalization, row-versus-column orientation, and the c-number-versus-anticommuting-operator map); Eq. (C2) is stated in the normalized Nieves–Pal c-number basis with the explicit Grassmann-exchange step, matching the companion paper's published Fierz appendix [1] and giving the γ-independent mean-field scalar-channel factor $G_s = -3\kappa/16$ of the companion's gap-equation convention. The Fierz rearrangement supplies only this channel factor; the full Sec. II contact operator carries the additional finite-Holst prefactor $\gamma^2/(1+\gamma^2) < 1$, which arises from the Cartan torsion elimination (not from the Fierz step; the elimination chain is carried in App. E) and which the companion's gap-equation bound conservatively omits because it can only reduce the coupling (equivalently, $G_s = -3\kappa/16$ is the $\gamma \to \infty$ limit) [1]. This coefficient set is independently verified by explicit 4×4 Dirac-matrix construction in both metric signatures and an exact Grassmann-algebra derivation of the operator row (the unique solution for identical fields); see the released verification artifact and its report listed in the Data and Code Availability statement. The released script `arxiv/scripts/fierz_lemma_check.py` independently checks the same convention's matrix involution and axial-row coefficients. None of this survey's conclusions depends on the convention-bound coefficients beyond this identity: the produced classes close on $\{SS, VV, AA, PP\}$ (the tensor channel is absent from the axial row), every coefficient is a dimensionless $O(1)$ rational, and the $\kappa \sim M_{\rm Pl}^{-2}$ prefactor is preserved exactly — which is all the projection lemma uses. Every produced structure lies in the closed set $\{SS, PP, VV, AA\}$ (with $\{V, A\}$ for the cross term); the tensor channel does not appear, and no operator escapes the enumerated basis. Because every coefficient in F is a dimensionless rational, the Fierz map is an $O(1)$ recombination of the *same* four ψ fields and therefore preserves the $\kappa \sim M_{\rm Pl}^{-2}$ prefactor exactly. This is the explicit Fierz-by-Fierz projection lemma: within single-species minimal ECH the generated four-fermion tower is Fierz-closed and uniformly $M_{\rm Pl}^{-2}$-suppressed, so the single-scale NDA ceiling of App. A that bounds one representative bounds the entire finite basis. (For multiple fermion species the Lorentz-class closure and the $M_{\rm Pl}$-power argument are unchanged—Fierz acts on Dirac structure, not flavor—and only the flavor-off-diagonal coefficient bookkeeping is added.) The only operators evading this bound require a *non-minimal* completion (a new light scale $\mu \ll M_{\rm Pl}$, trace/tensor torsion irreps, or a dynamical Chern–Simons field), which is the stated scope boundary of the no-go.

Scope of the lemma (what it establishes and what it does not).—The Fierz lemma establishes basis-completeness of the *parity-odd four-fermion contact sector* at $M_{\rm Pl}$-power-counting within the minimal ECH field content (tetrad, algebraic torsion via Cartan constraint, Holst term, and minimally-coupled fermion). It does *not* enumerate: derivative four-fermion terms suppressed by additional powers of momentum/$M_{\rm Pl}$; curvature-torsion mixed invariants built from higher-order contractions of the Riemann tensor and spin current; chiral/flavor structures involving independent fermion species beyond the single-species minimal content; dynamical topological coefficients arising from a promoted Immirzi field; or non-minimal torsion irreps (trace-vector and tensor irreps) that appear only when the minimal coupling assumption is relaxed. The claim is scoped to the minimal-ECH field content at the stated power-counting order; it is not a complete operator-level no-go over all diffeomorphism-invariant operators in the full gravitational EFT space.

Appendix D: Self-Contained Statement and Proof of the Perturbation-Transparency Theorem (B14)

So that the catalog's sole Tier-I leg can be refereed from this manuscript, we carry here, faithfully from the companion paper's zero-spin-branch analysis [1], the statement and proof of the perturbation-transparency theorem. The tensor-sector extension, the explicit second-order Holst-term verification, and the discussion of what would break the transparency are given in full in the companion; nothing below is new to this survey.

Statement.—Consider the classical ECH action with an invertible tetrad, canonical scalar matter, and a real finite nonzero constant γ, on a local patch with matched background, initial, and boundary data (standard falloff conditions, boundary contribution to the first-order variation set to zero). Solve the algebraic connection equation before expanding in perturbations. Then the reduced action on the zero-spin branch is the Einstein–scalar action, so scalar equations and tensor evolution operators coincide with those of general relativity at every perturbative order for the same background and boundary data. Equality of right- and left-helicity *solutions*, rather than of their evolution operators, additionally requires parity-symmetric initial data. The complex singular values $\gamma = \pm i$, nontrivial global or topological sectors, nonstandard boundary terms, quantum loops or anomalies, fermion sources, non-minimal matter, a dynamical Immirzi field, and propagating torsion are outside this statement. This is equality *after* solving the Cartan equation — an on-connection-shell equality of local classical reduced actions — not an off-shell equality of the original first-order actions; its all-orders content does not come from an order-by-order calculation, because the Cartan constraint is algebraic and the Bianchi identity used below is pointwise.

Proof.—(1) *Zero spin density.* A canonical scalar field has zero spin density. (2) *Zero torsion.* The Holst-modified connection equation acts through an algebraic bivector operator whose inverse exists for real finite nonzero γ (because $1+\gamma^2>0$); the zero scalar source therefore gives $e^{[I}\wedge T^{J]}=0$, whose invertible-tetrad kernel is trivial: $T^I=0$. This holds at every classical field configuration in the stated local domain, hence before any perturbative expansion. (3) *The connection reduces to Levi-Civita*, ${\Gamma^\lambda}_{\mu\nu} = {\mathring{\Gamma}^\lambda}_{\mu\nu}$. (4) *The Holst term vanishes by the first Bianchi identity.* Evaluated on the Levi-Civita connection the Holst term reduces to $\frac{1}{2\gamma}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}(\mathring{\Gamma})$ (retaining the explicit γ^{-1} prefactor of Sec. II), which vanishes identically by the algebraic Bianchi identity $R_{\mu[\nu\rho\sigma]}=0$ — the same pointwise identity verified symbolically as Check A [Eq. (A4)]. The Holst dual contraction is therefore identically zero on the torsion-free branch (not merely a boundary term) and contributes nothing to the action at any order. No total-derivative argument is a load-bearing step in this proof; the operative statement is the pointwise algebraic Bianchi identity in Step 4. ■

Consequences carried into the catalog: the Holst sector and the Barbero–Immirzi parameter decouple from all scalar and tensor perturbation observables for canonical scalar matter (B14, Branch H, constraining R1–R4), and in particular minimal ECH produces no helicity-dependent tensor propagation from parity-symmetric initial data — the observational content whose channel-level consequence B8 records.

Appendix E: Companion-Imported Tier-II Inputs Carried Self-Contained: Contact Coefficient and R1 Benchmark

Following the same faithful-extraction convention as Appendix D, this appendix carries, from the companion paper's published derivations [1], the two remaining load-bearing companion inputs of this survey: the torsion-elimination chain that fixes the minimal contact operator and its $-(3\kappa/16)[\gamma^2/(1+\gamma^2)]$ coefficient (Sec. II), and the R1 finite-density benchmark arithmetic quoted in Table II. Nothing below is new to this survey; the derivations follow the companion, which in turn follows Freidel, Minic, and Takeuchi [7].

1. Torsion elimination and the contact coefficient

Define the internal dual on Lorentz bivectors by $(\star X)_{IJ} = \frac{1}{2}{\epsilon_{IJ}}^{KL}X_{KL}$, so $\star^2=-\mathbb{1}$. Acting on an antisymmetric bivector, the Holst-modified Cartan operator of the first-order ECH action and its inverse are

$$\mathcal{Q}_\gamma = \star + \gamma^{-1}\mathbb{1}, \qquad \mathcal{Q}_\gamma^{-1} = \frac{\gamma^2}{1+\gamma^2}\left(\gamma^{-1}\mathbb{1} - \star\right), \quad \text{(E1)}$$

verified by direct multiplication; the inverse exists for every real finite nonzero γ because $1+\gamma^2 > 0$ — the same invertible bivector operator whose triviality of kernel drives Step 2 of Appendix D. Writing the Lorentz-connection source three-form as $\mathcal{J}^{IJ} \propto \delta S_{\mathrm{matter}}/\delta\omega_{IJ}$, the connection variation reads $\mathcal{Q}_\gamma(e^{[I} \wedge T^{J]}) = \mathcal{J}^{IJ}$: an algebraic (non-dynamical) equation. For minimal (totally antisymmetric) Dirac coupling, whose spin current is dual to the axial current [$S^{IJK} = \frac{1}{4}\epsilon^{IJKL}J_{5L}$, the Sec. II convention; in the Einstein–Cartan limit the constraint reads $T^{IJK} = \kappa S^{IJK} = (\kappa/4)\epsilon^{IJKL}J_{5L}$], Freidel, Minic, and Takeuchi solve this equation explicitly [their Eq. (17)], obtaining the contorsion

$$e_I{}^\mu C_{\mu JK} = 4\pi G\,\frac{\gamma^2}{1+\gamma^2}\left(\frac{1}{2}\epsilon_{IJKL}J_5^L - \frac{1}{\gamma}\,\eta_{I[J}J^5_{K]}\right), \tag{E2}$$

and their direct back-substitution into the action [their Eq. (23)] gives $\mathcal{L}_{\mathrm{int}} = -(3/2)\pi G\,[\gamma^2/(1+\gamma^2)]\,J_5^2$ for minimal coupling. The normalization bridge to this survey's $\kappa = 8\pi G$ is

$$4\pi G = \frac{\kappa}{2}, \qquad -\frac{3}{2}\pi G = -\frac{3\kappa}{16}, \tag{E3}$$

so the eliminated-torsion contact operator is

$$\mathcal{L}_{4\psi} = -\frac{3\kappa}{16}\,\frac{\gamma^2}{1+\gamma^2}\,(J_5^I J_{5I}), \tag{E4}$$

exactly the Sec. II operator. The pure Einstein–Cartan coefficient $-(3\kappa/16)$ is recovered for $\gamma \to \infty$; because $\gamma^2/(1+\gamma^2) < 1$ for real finite γ can only reduce the coupling, the companion's gap-equation convention retains the maximal magnitude $G_s = -3\kappa/16$ — the γ-independent channel factor of Appendix C [1]. A vector–axial term is not generated by minimal coupling; it requires a non-minimal fermion coupling, outside the declared field content.

2. The R1 finite-density benchmark

For a specified homogeneous fermion number density n_ψ the companion defines the coefficient-one dimensional benchmark $\rho_{4f}^{(\mathrm{dim})} \equiv \kappa n_\psi^2 = 8\pi n_\psi^2/M_{\mathrm{Pl}}^2$ (mass dimension $+4$, since $[n_\psi] = +3$); the actual axial–axial operator Eq. (E4) adds the factor $3/16$. Using $\hbar c = 1.9733 \times 10^{-5}\,\mathrm{eV\,cm}$ and $M_{\mathrm{Pl}} = 1.2209 \times 10^{28}\,\mathrm{eV}$,

$$\begin{aligned}\kappa n_\psi^2 &\simeq 1.0 \times 10^{-79}\left(\frac{n_\psi}{100\,\mathrm{cm}^{-3}}\right)^2 \mathrm{eV}^4,\\ \frac{\kappa n_\psi^2}{\rho_\Lambda} &\simeq 3.6 \times 10^{-69}\left(\frac{n_\psi}{100\,\mathrm{cm}^{-3}}\right)^2,\end{aligned} \tag{E5}$$

with the companion's normalization $\rho_\Lambda \approx (2.3\,\mathrm{meV})^4 \approx 2.8 \times 10^{-11}\,\mathrm{eV}^4$ — about 68.4 orders below ρ_Λ at the deliberately elevated ISM-like normalization $100\,\mathrm{cm}^{-3}$ (neither a cosmological-density estimate nor a preferred state); including the $3/16$ coefficient gives $1.9 \times 10^{-80}\,\mathrm{eV}^4 = 6.7 \times 10^{-70}\,\rho_\Lambda$. Under the $\rho_\Lambda = (2.25\,\mathrm{meV})^4$ input of Appendix A the same κn_ψ^2 evaluates to 3.9×10^{-69}, identical at the order-of-magnitude precision quoted (Sec. II). As in the companion, this comparison is a coefficient-scale illustration only: the number density does not determine the state-dependent coincident composite $\langle J^5 \cdot J^5\rangle$ without specifying the ensemble, spin polarization, and a renormalized contact prescription, and no coherent order parameter, vacuum expectation value, or equation of state is inferred from it.

[1] H. Golden, Algebraic Cartan Elimination in Minimal Einstein–Cartan–Holst Gravity: Spin-Sourced Contact and Zero-Spin Scalar Branches, Zenodo 10.5281/zenodo.21481838 (2026), Companion paper (Paper I A), publicly archived on Zenodo under version DOI 10.5281/zenodo.21481838 (concept DOI 10.5281/zenodo.21481837), deposited 2026 July 22 under CC-BY-4.0. Public permanent archive of the manuscript and its exact source; not an arXiv preprint and not peer reviewed.

[2] I. L. Shapiro and P. M. Teixeira, Quantum Einstein-Cartan theory with the Holst term, Classical and Quantum Gravity **31**, 185002 (2014), arXiv:1402.4854 [gr-qc].

[3] D. Benedetti and S. Speziale, Perturbative running of the Immirzi parameter, J. Phys. Conf. Ser. **360**, 012011 (2012), arXiv:1111.0884 [hep-th].

[4] A. Ashtekar and P. Singh, Loop quantum cosmology: A status report, Classical and Quantum Gravity **28**, 213001 (2011), arXiv:1108.0893 [gr-qc].

[5] A. Ghosh and P. Mitra, An improved estimate of black hole entropy in the quantum geometry approach, Phys. Lett. B **616**, 114 (2005), arXiv:gr-qc/0411035 [gr-qc].

[6] S. Holst, Barbero's Hamiltonian derived from a generalized Hilbert-Palatini action, Physical Review D **53**, 5966 (1996), arXiv:gr-qc/9511026 [gr-qc].

[7] L. Freidel, D. Minic, and T. Takeuchi, Quantum gravity, torsion, parity violation and all that, Physical Review D **72**, 104002 (2005), arXiv:hep-th/0507253 [hep-th].

[8] S. Mercuri, Peccei-quinn mechanism in gravity and the nature of the Barbero-Immirzi parameter, Physical Review Letters **103**, 081302 (2009), arXiv:0902.2764 [gr-qc].

[9] G. Date, R. K. Kaul, and S. Sengupta, Topological interpretation of Barbero-Immirzi parameter, Phys. Rev. D **79**, 044008 (2009), arXiv:0811.4496 [gr-qc].

[10] D. Benedetti and S. Speziale, Perturbative quantum gravity with the Immirzi parameter, JHEP **2011** (6), 107, arXiv:1104.4028 [hep-th].

[11] J. R. Eskilt and E. Komatsu, Improved constraints on cosmic birefringence from the WMAP and Planck cosmic microwave background polarization data, Phys. Rev. D **106**, 063503 (2022), arXiv:2205.13962 [astro-ph.CO].

[12] Y. Minami and E. Komatsu, New extraction of the cosmic birefringence from the Planck 2018 polariza-

tion data, Physical Review Letters **125**, 221301 (2020), arXiv:2011.11254 [astro-ph.CO].

[13] P. Diego-Palazuelos and E. Komatsu, Cosmic birefringence from the Atacama Cosmology Telescope data release 6, (2025), arXiv preprint; no journal reference at time of writing., arXiv:2509.13654 [astro-ph.CO].

[14] H. Golden, namaster-proof: Exact pseudo-C_ℓ window inference and content-bound validation for reproducible spin-2 analyses, Zenodo 10.5281/zenodo.21481842 (2026), Companion paper (Paper I B), publicly archived on Zenodo under version DOI 10.5281/zenodo.21481842 (concept DOI 10.5281/zenodo.21481841), deposited 2026 July 22 under CC-BY-4.0. Public permanent archive of the manuscript and its exact source; not an arXiv preprint and not peer reviewed. The software it describes is archived separately at 10.5281/zenodo.21481753.

[15] S. M. Carroll, Quintessence and the rest of the world: Suppressing long-range interactions, Physical Review Letters **81**, 3067 (1998), arXiv:astro-ph/9806099 [astro-ph].

[16] Y.-F. Cai, E. N. Saridakis, M. R. Setare, and J.-Q. Xia, Quintom Cosmology: Theoretical implications and observations, Phys. Rept. **493**, 1 (2010), arXiv:0909.2776 [hep-th].

[17] T. Kimura, Divergence of axial-vector current in the gravitational field, Progress of Theoretical Physics **42**, 1191 (1969).

[18] R. Delbourgo and A. Salam, The gravitational correction to PCAC, Physics Letters B **40**, 381 (1972).

[19] H. T. Nieh and M. L. Yan, An identity in riemann–cartan geometry, J. Math. Phys. **23**, 373 (1982).

[20] N. J. Popławski, Cosmology with torsion: An alternative to cosmic inflation, Physics Letters B **694**, 181 (2010), arXiv:1007.0587 [astro-ph.CO].

[21] G. Buchalla, O. Catà, and C. Krause, On the Power Counting in Effective Field Theories, Phys. Lett. B **731**, 80 (2014), arXiv:1312.5624 [hep-ph].

[22] I. Brivio and M. Trott, The Standard Model as an Effective Field Theory, Phys. Rept. **793**, 1 (2019), arXiv:1706.08945 [hep-ph].

[23] G. Isidori, F. Wilsch, and D. Wyler, The Standard Model effective field theory at work, Rev. Mod. Phys. **96**, 015006 (2024), arXiv:2303.16922 [hep-ph].

[24] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, 1980).

[25] J. F. Nieves and P. B. Pal, Generalized fierz identities, Am. J. Phys. **72**, 1100 (2004), arXiv:hep-ph/0306087 [hep-ph].